Wireless Network Coding

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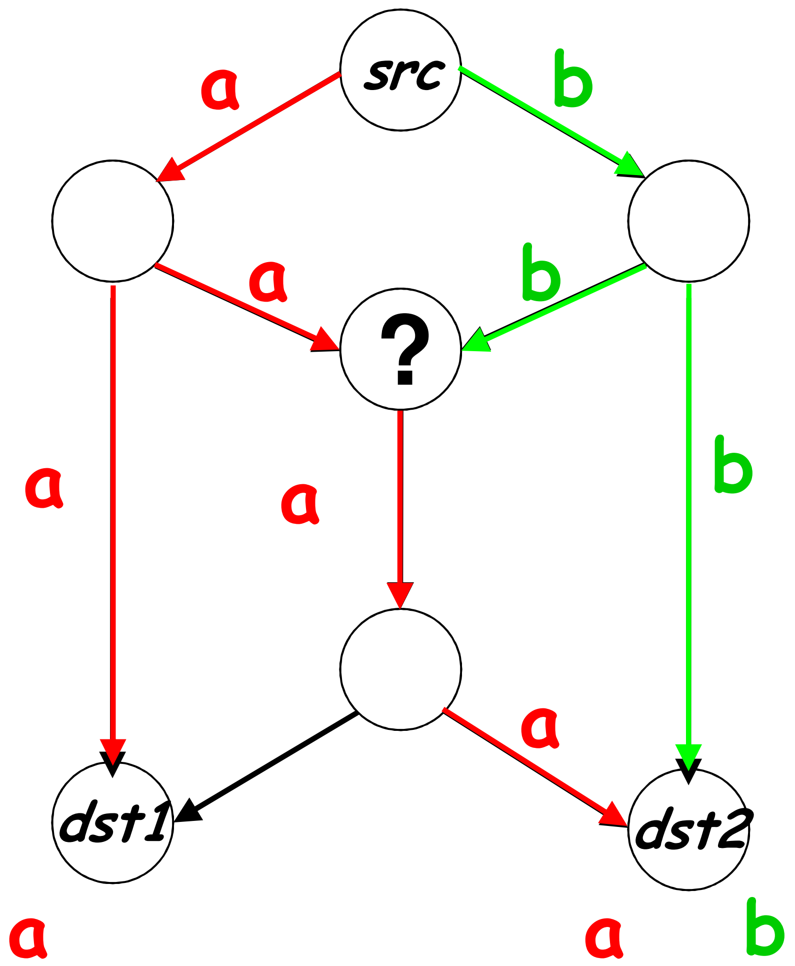
What is Network Coding?

- Generalization of store and forward networks
- Routers mix/code packets' content before forwarding

The Butterfly Example

Source wants to multicasts *a* and *b* to both destinations

Link capacity is 1 message/second



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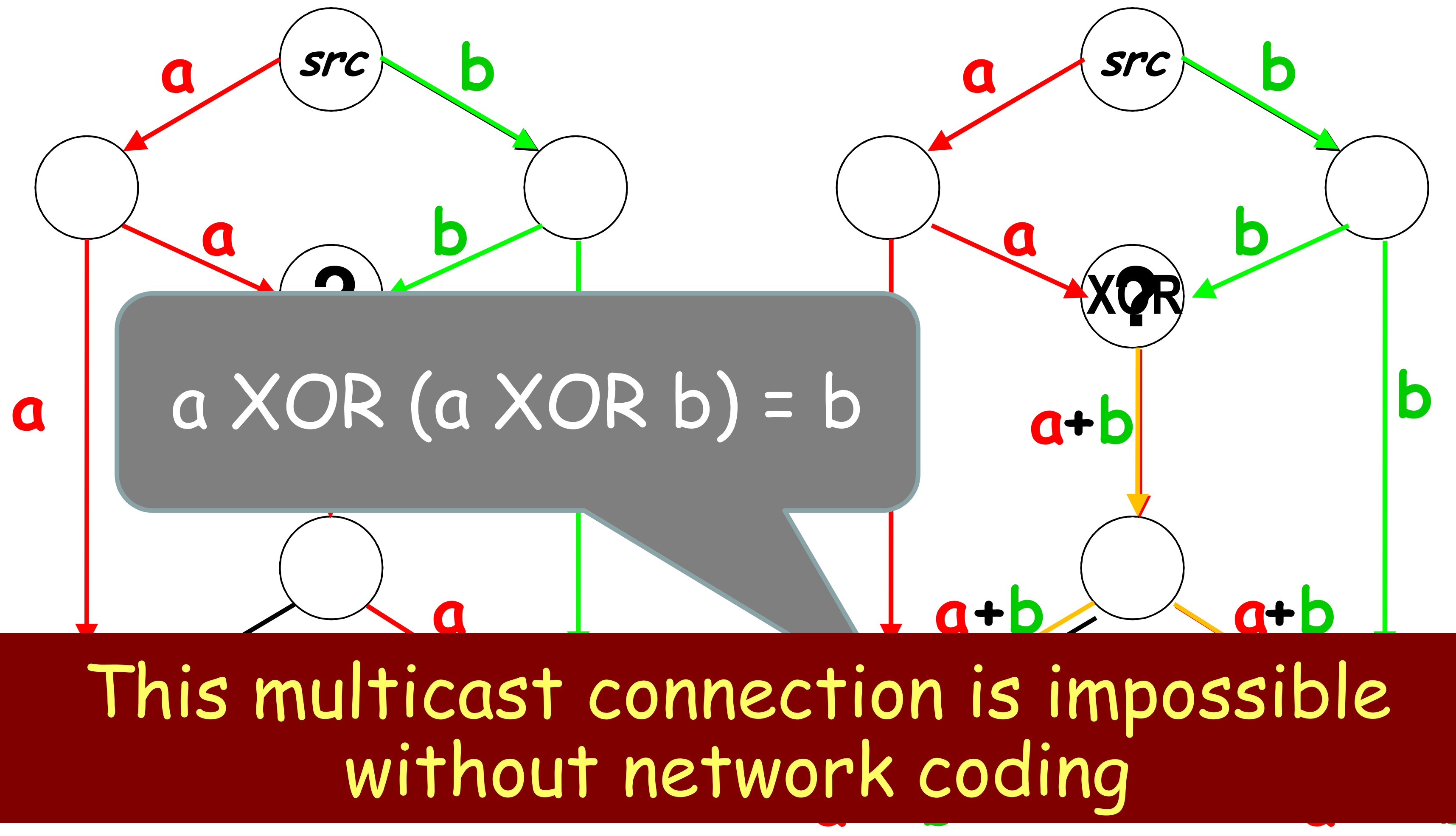


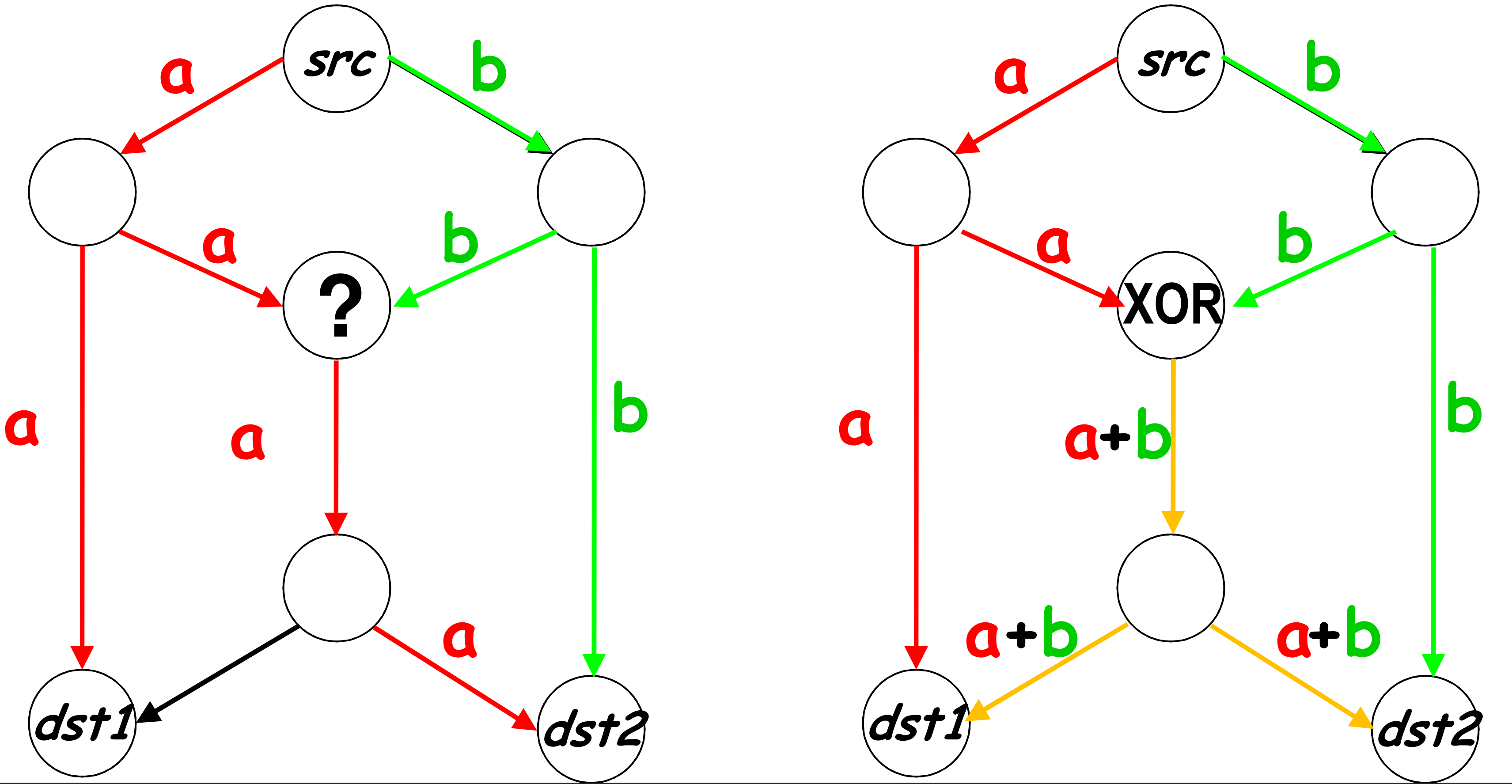
a XOR (a XOR b) = b

This multicast connection is impossible without network coding

What is Network Coding Good for?

- Throughput

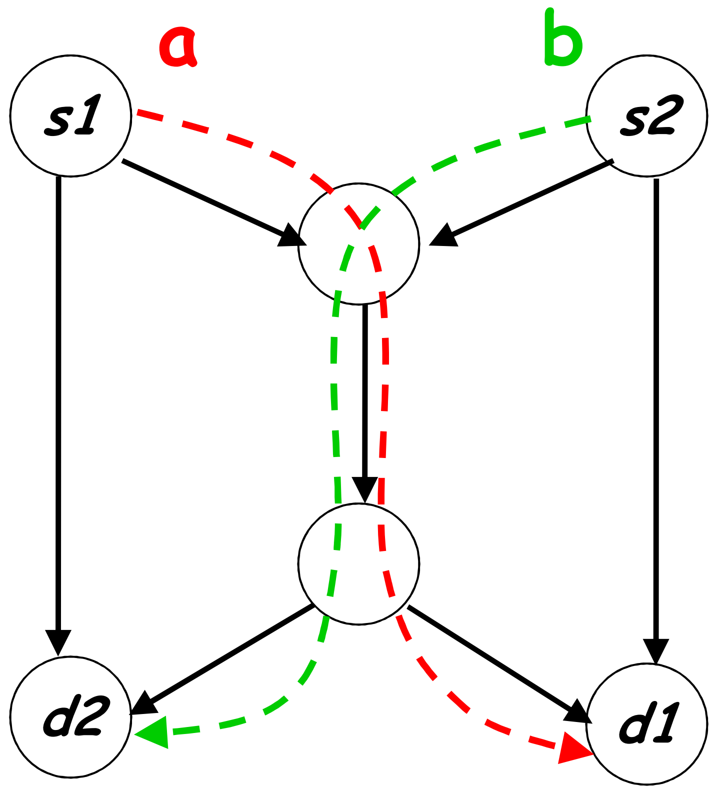
Improves Multicast Throughput



Without network coding, multicast throughput is 1.5
With network coding, multicast throughput is 2

Improves Unicast Throughput

s1 wants to send *a* to d1, and s2 wants to send *b* to d2



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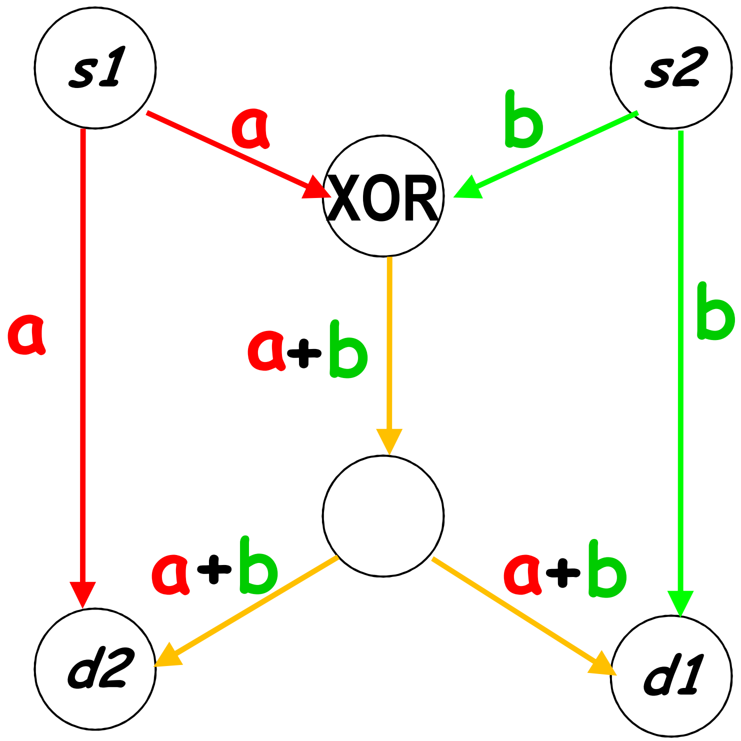


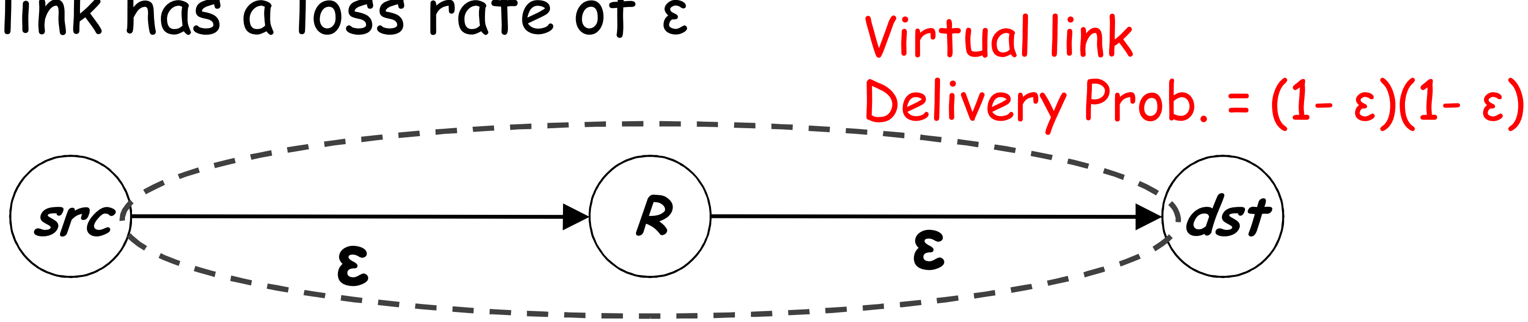
Without net-coding, avg. per flow throughput is 0.5
With net-coding, avg. per-flow throughput is 1

What is Network Coding Good for?

- Throughput
- Robustness

Robustness to Packet Loss

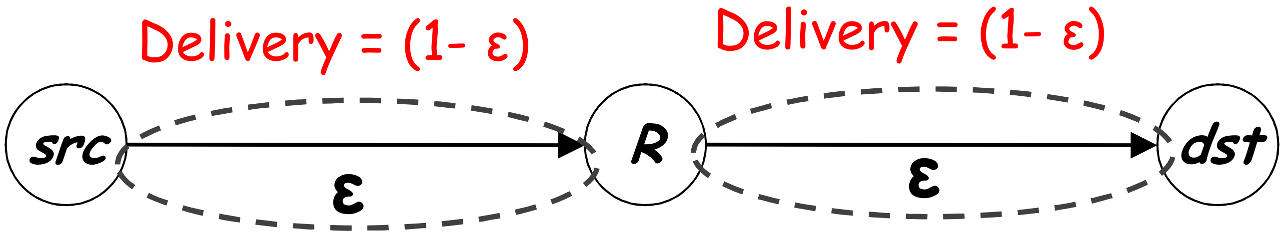
Each link has a loss rate of ε



- With source coding, the maximum rate is (1- ε)(1- ε)
  - E.g., ε=0.05; of each 100 packets 10% should be redundancy → data rate = 0.9
- With network coding, the maximum rate is (1- ε)

Robustness to Packet Loss

Each link has a loss rate of ε



- With source coding, the maximum rate is (1- ε)(1- ε)
  - E.g., ε=0.05; of each 100 packets 10% should be redundancy → data rate = 0.9
- With network coding, the maximum rate is (1- ε)
  - E.g., ε= 0.05; of each 100 packets only 5% should be redundancy →data rate =0.95

Robustness to Randomness

Coupon Collector Problem

- Problem: *n* unique coupons; a collector samples randomly
- Without coding
  - Need a sample size of about *nlog(n)* to collect all unique coupons
- With random coding
  - Need *n* samples to collect all unique coupons
  - e.g., 3 coupons c1, c2 and c3

$$Y_1 = c_1 + 3\ c_2 + 4\ c_3$$
$$Y_2 = 5\ c_1 + c_2 + 7c_3$$
$$Y_3 = c_1 + c_2$$

Two Types of Network Coding

Inter-flow

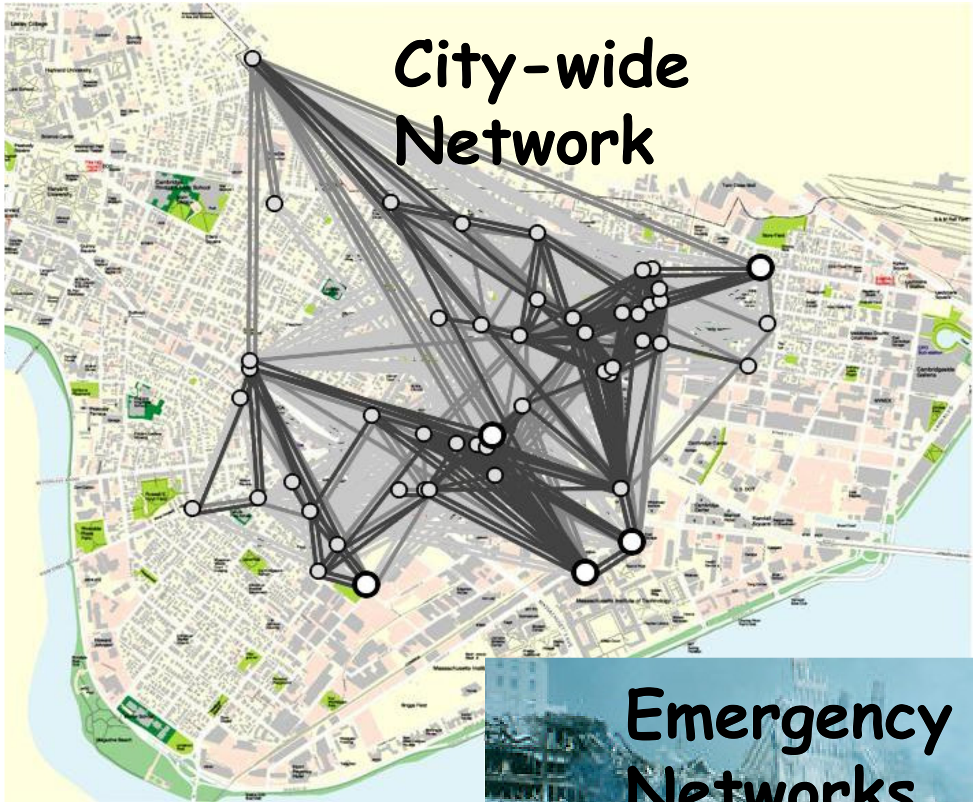
- Codes packets across connections
- Increases Throughput
- Mainly Unicast

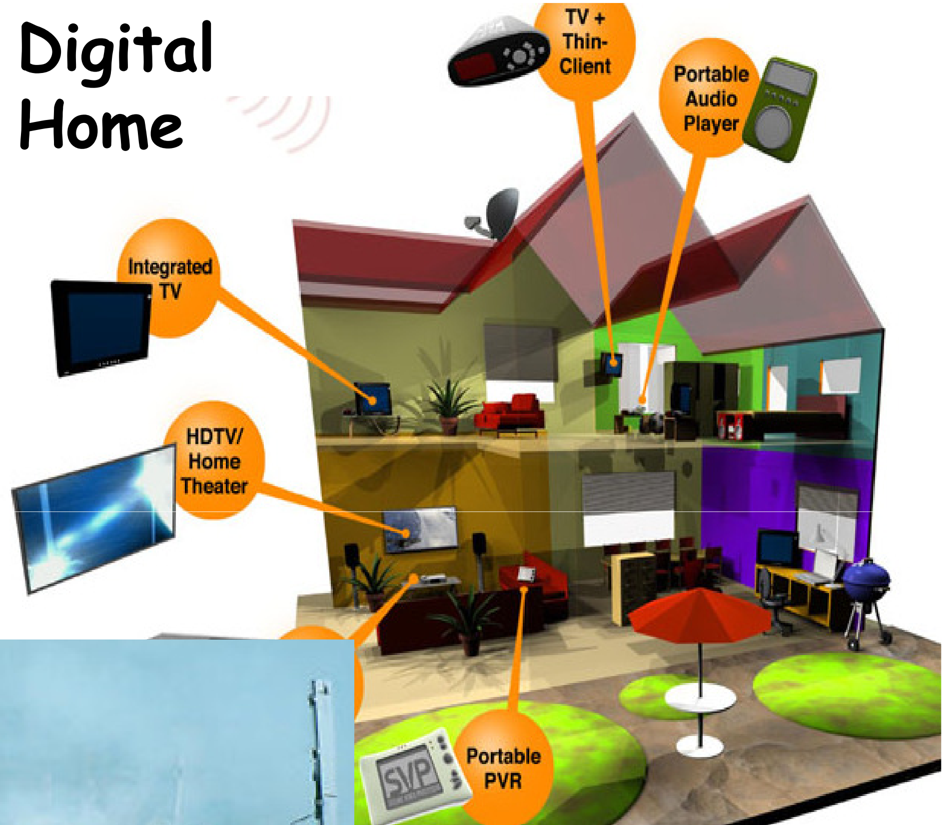
Intra-flow

- Codes packets within a connection
- Robustness to packet loss
- Mainly multicast

COPE
An Example of Inter-flow Network Coding

Increased Demands for Wireless Networks

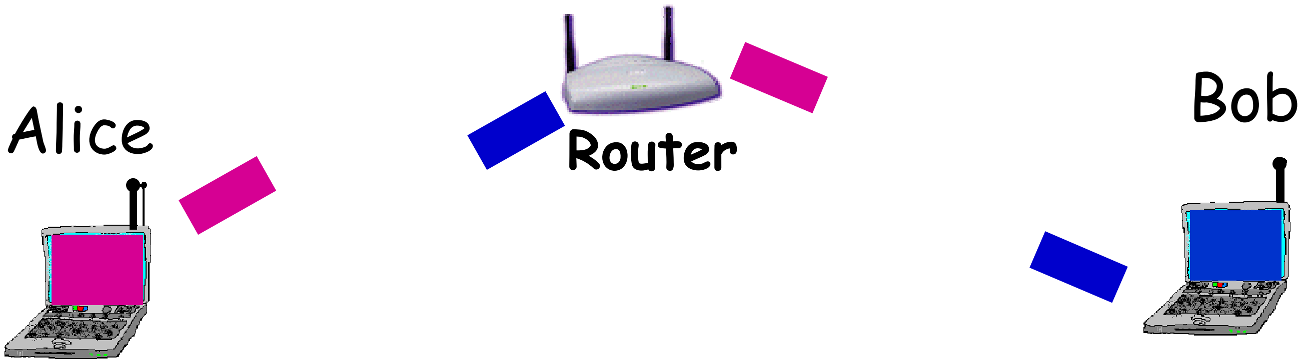




But, wireless networks struggle with low throughput, particularly in dense deployments

Can network coding help?

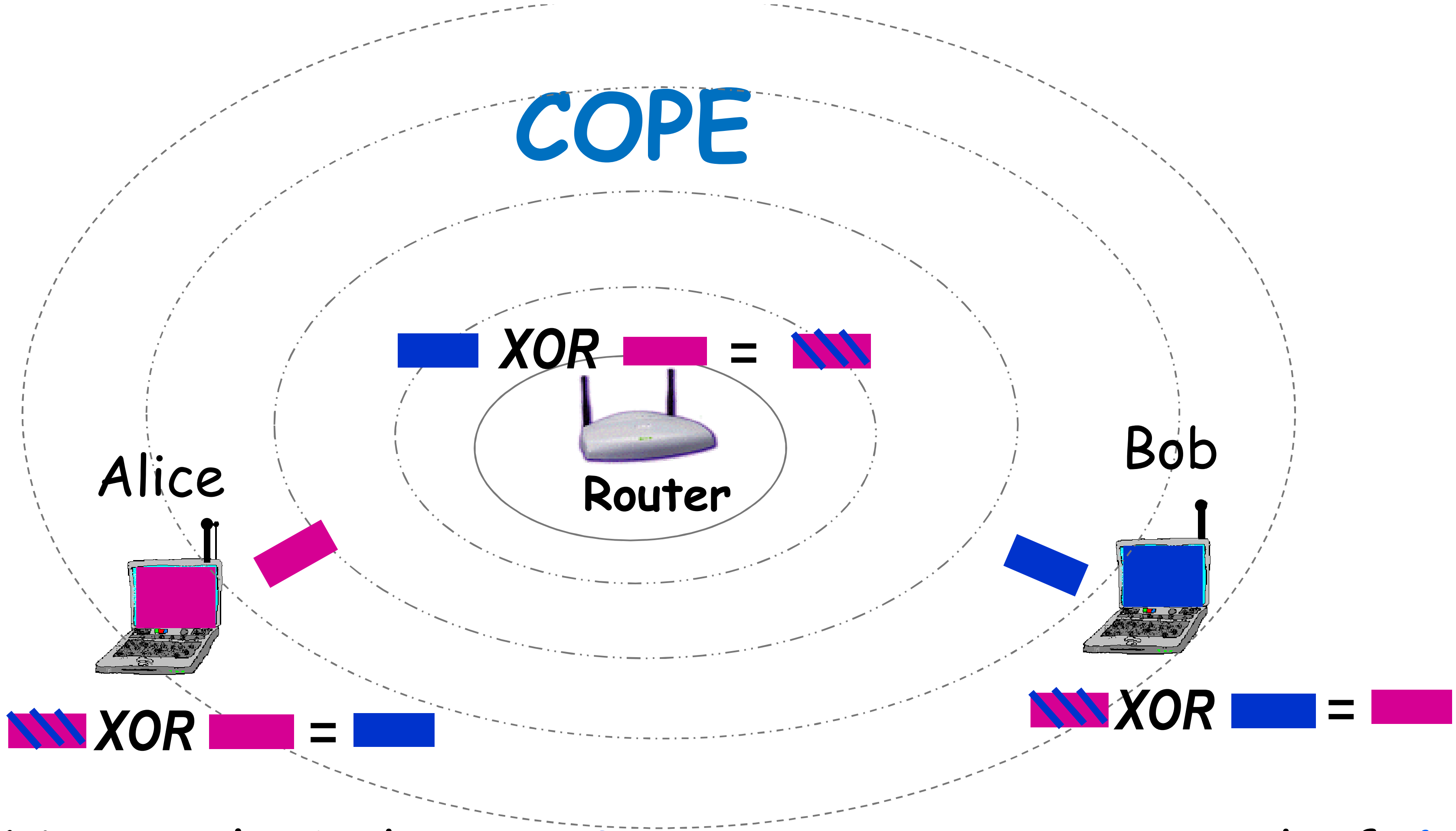
Current Approach



Current Approach → Requires 4 transmissions

But can we do better?

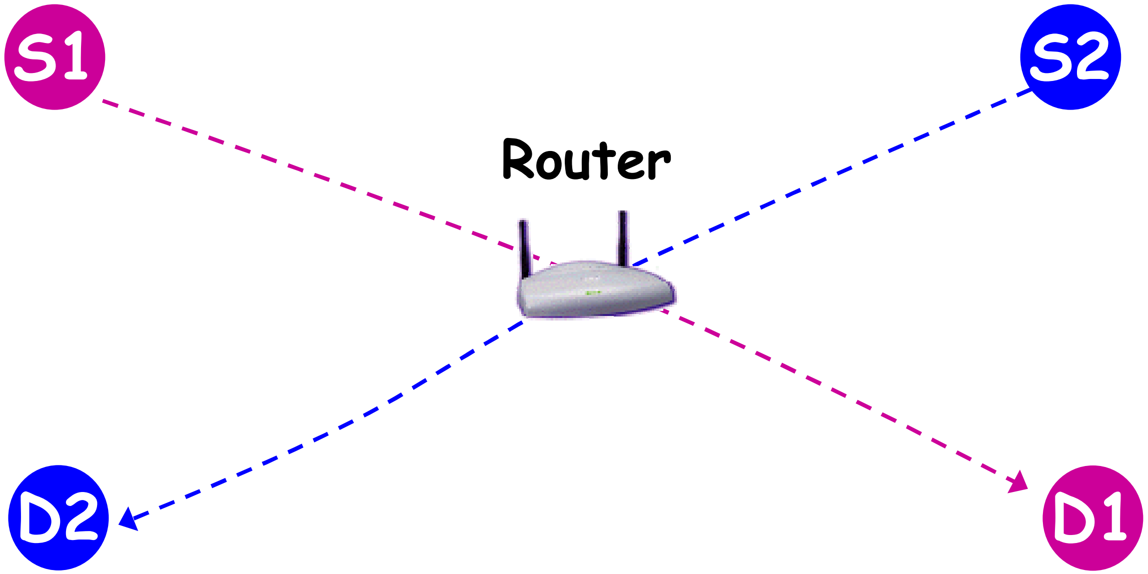
COPE



Network Coding → 3 transmissions instead of 4

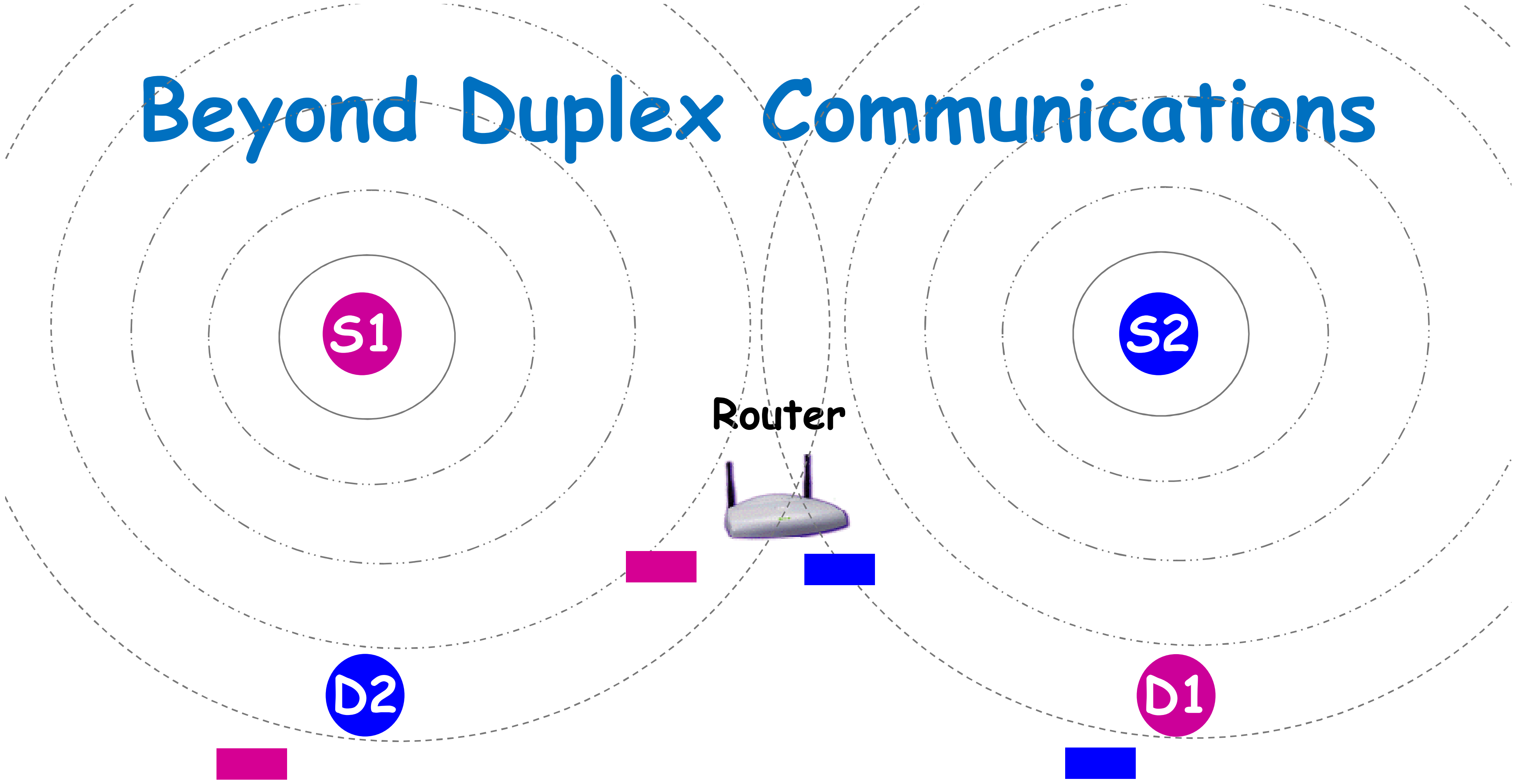
Network Coding Increases Throughput

Beyond Duplex Communications



Two communications that intersect at a router

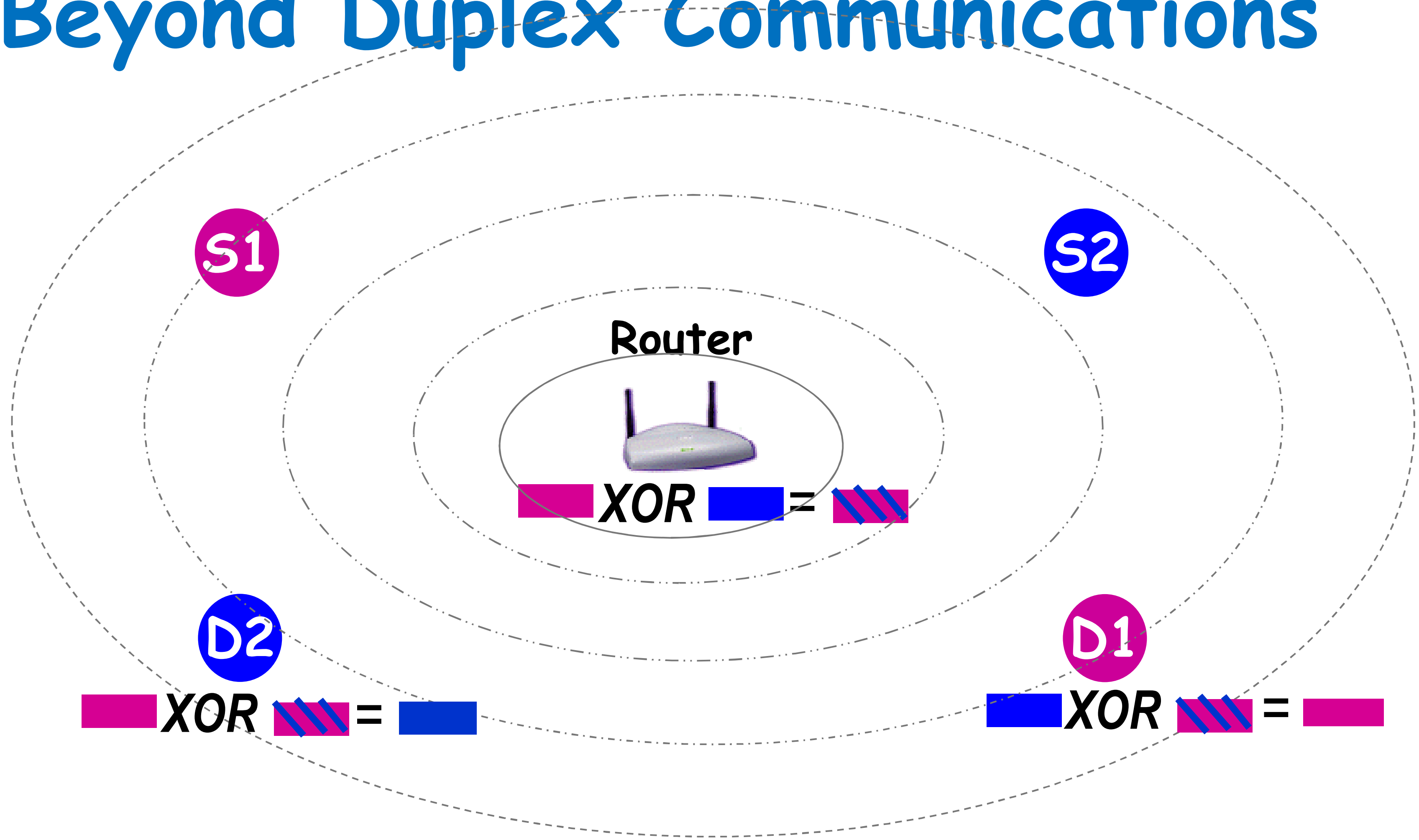
Beyond Duplex Communications



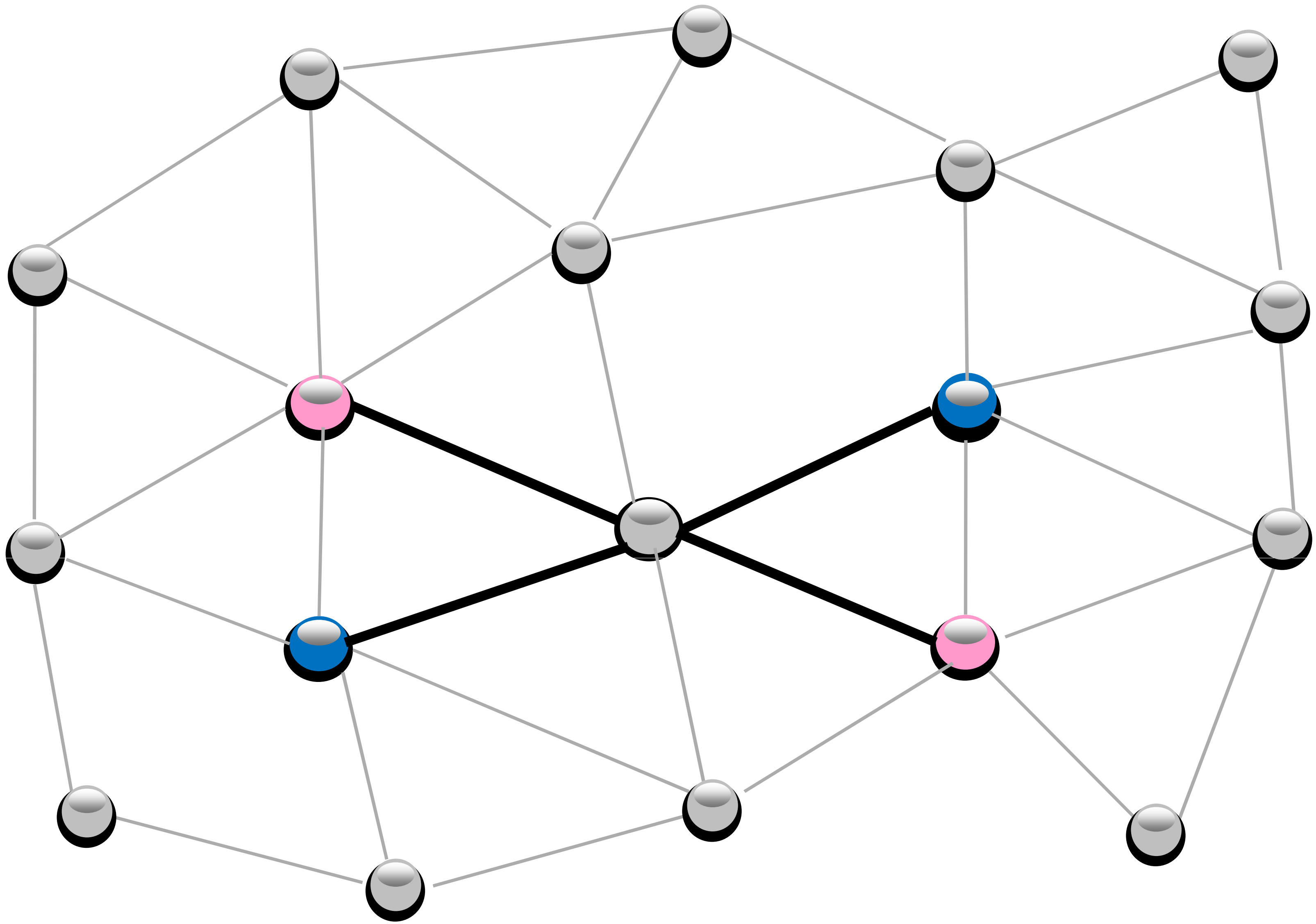
D2 overhears packet

D1 overhears packet

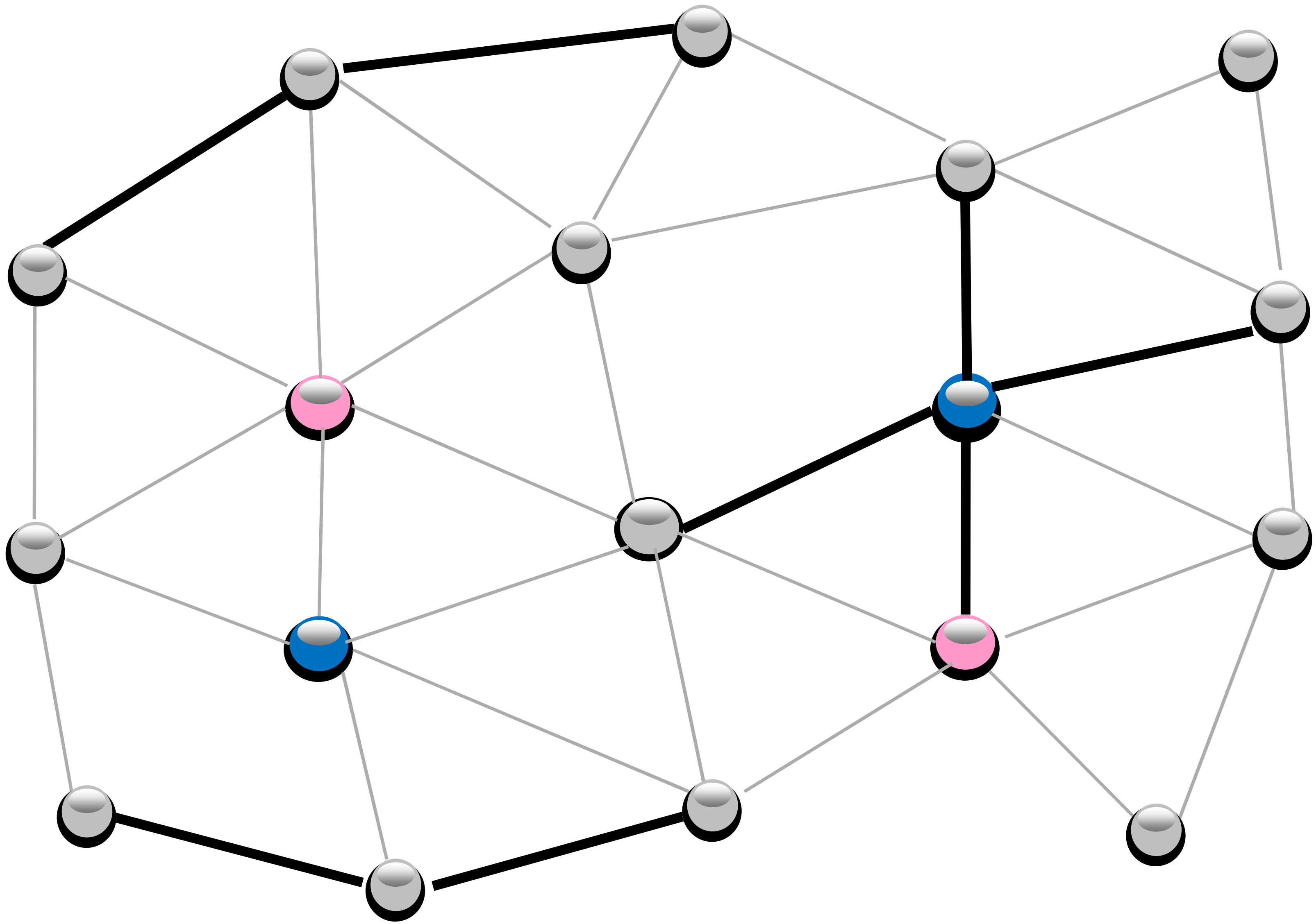
Beyond Duplex Communications



3 transmissions instead of 4 → Higher Throughput



Generalizes to arbitrary networks



Generalizes to arbitrary networks

Differences from Traditional Wireless Networks

- Embrace the broadcast nature of wireless
  - Dispose of the point-to-point abstraction

- Routers mix bytes across packets, then forward them → Network Coding

Exploit Broadcast - Snoop

- Nodes snoop on all packets
- A node stores all heard packets for a limited time

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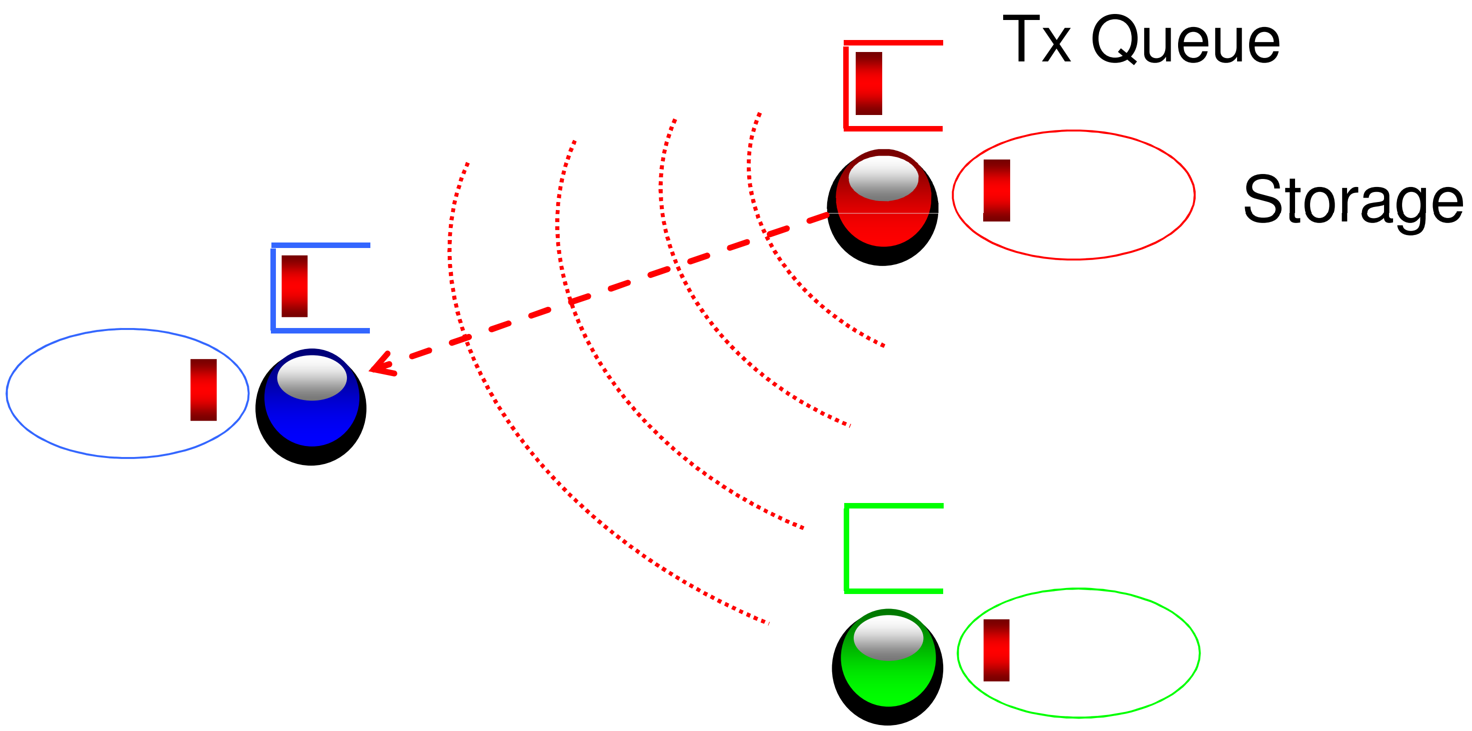
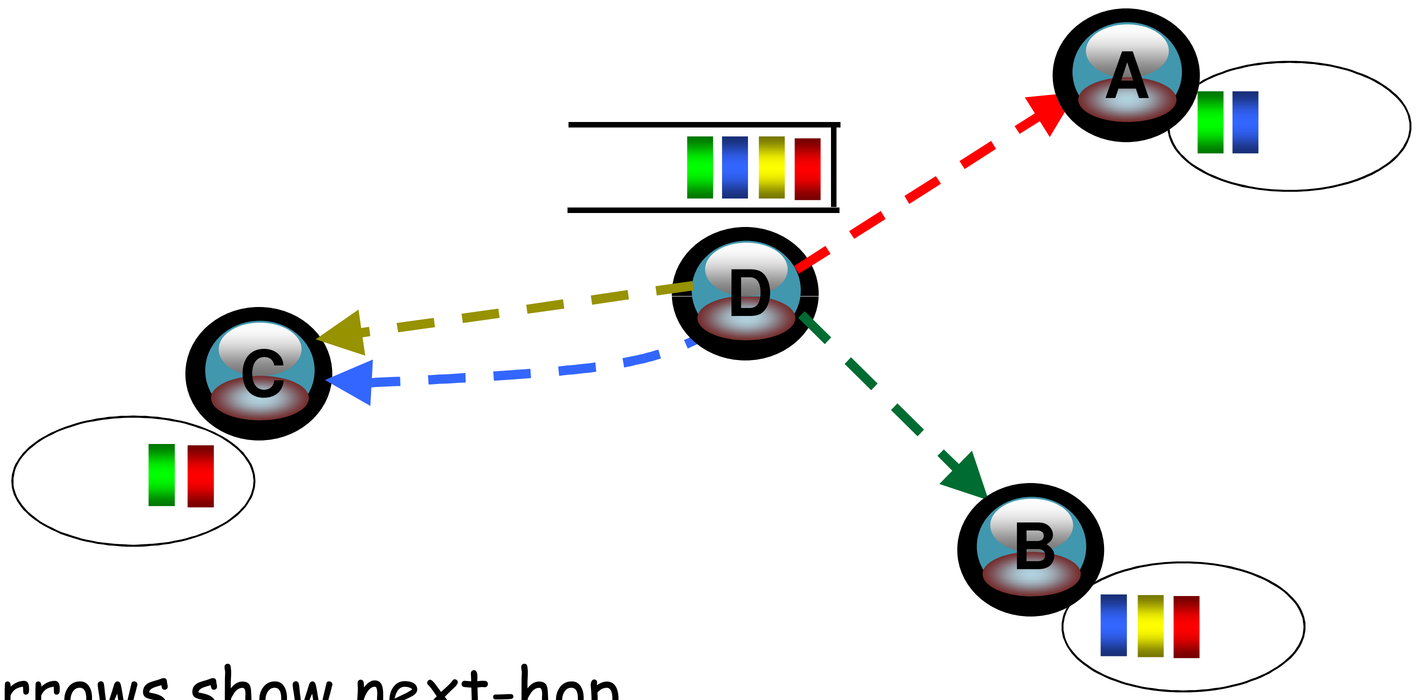


Routers Code Packets

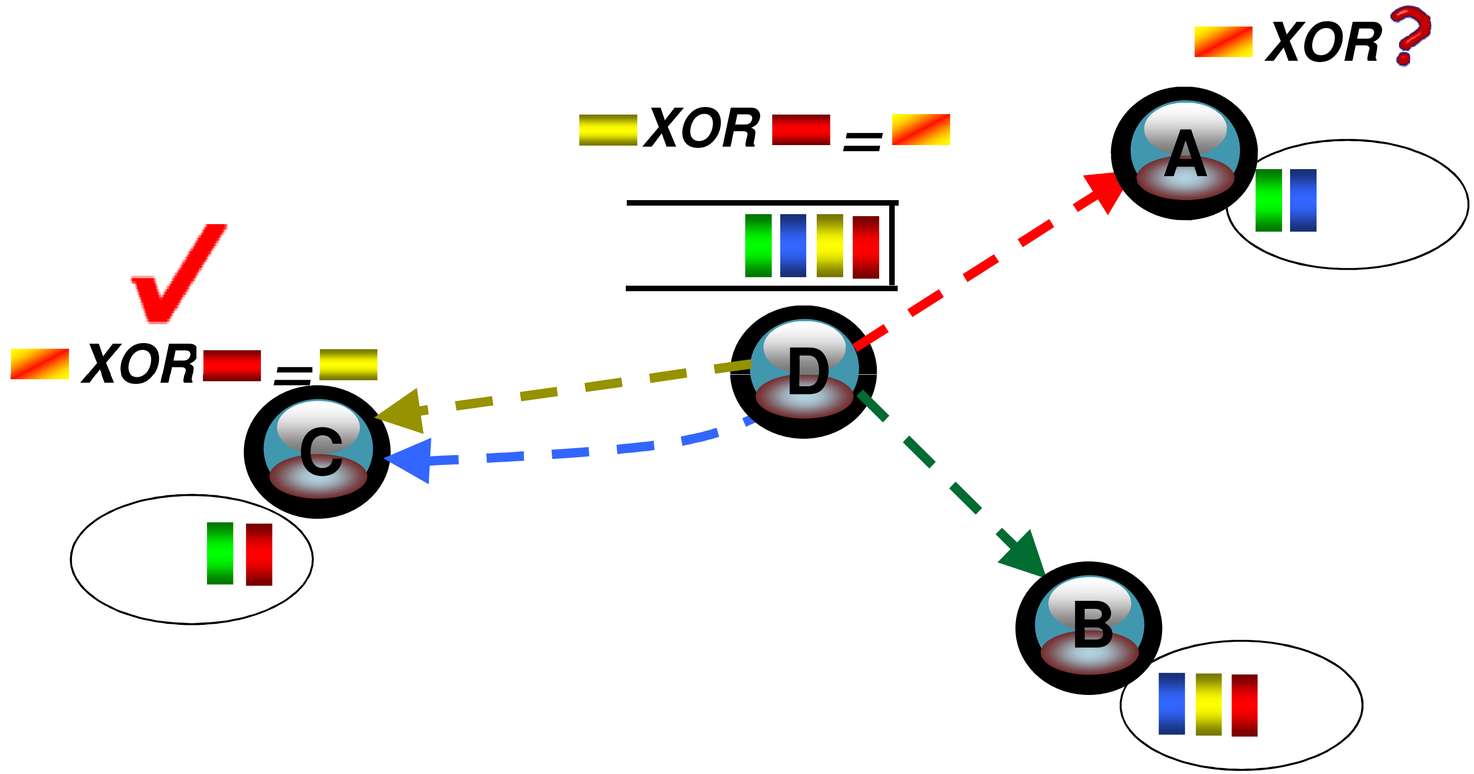
- To send packet *p* to neighbor *A*, XOR *p* with packets already known to *A*
  - Thus, *A* can decode
- But how can multiple neighbors benefit from a single transmission?

Which Packets to Code Together?



Arrows show next-hop

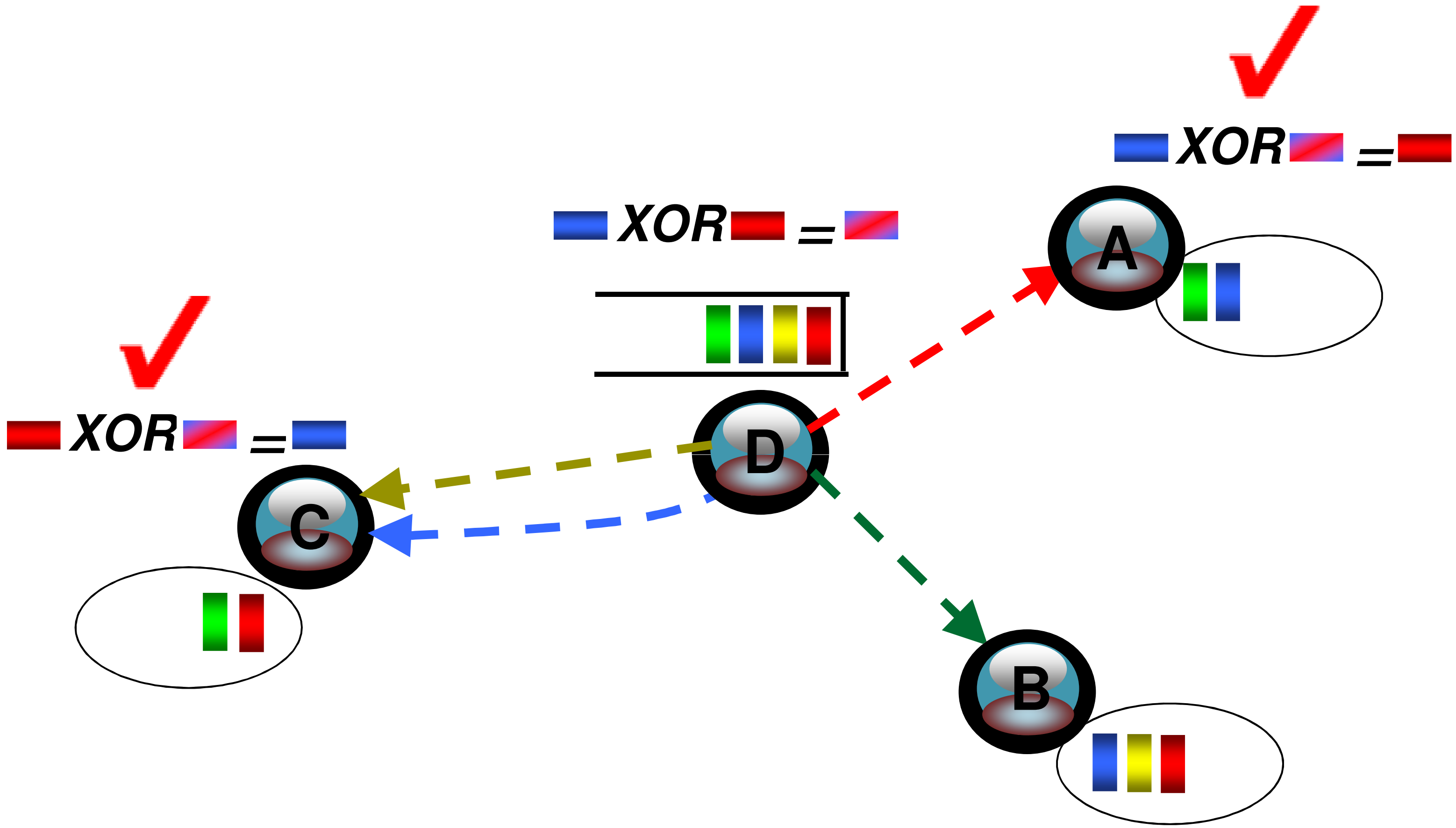
Which Packets to Code Together?



Bad Coding

Only one neighbor benefits from one transmission

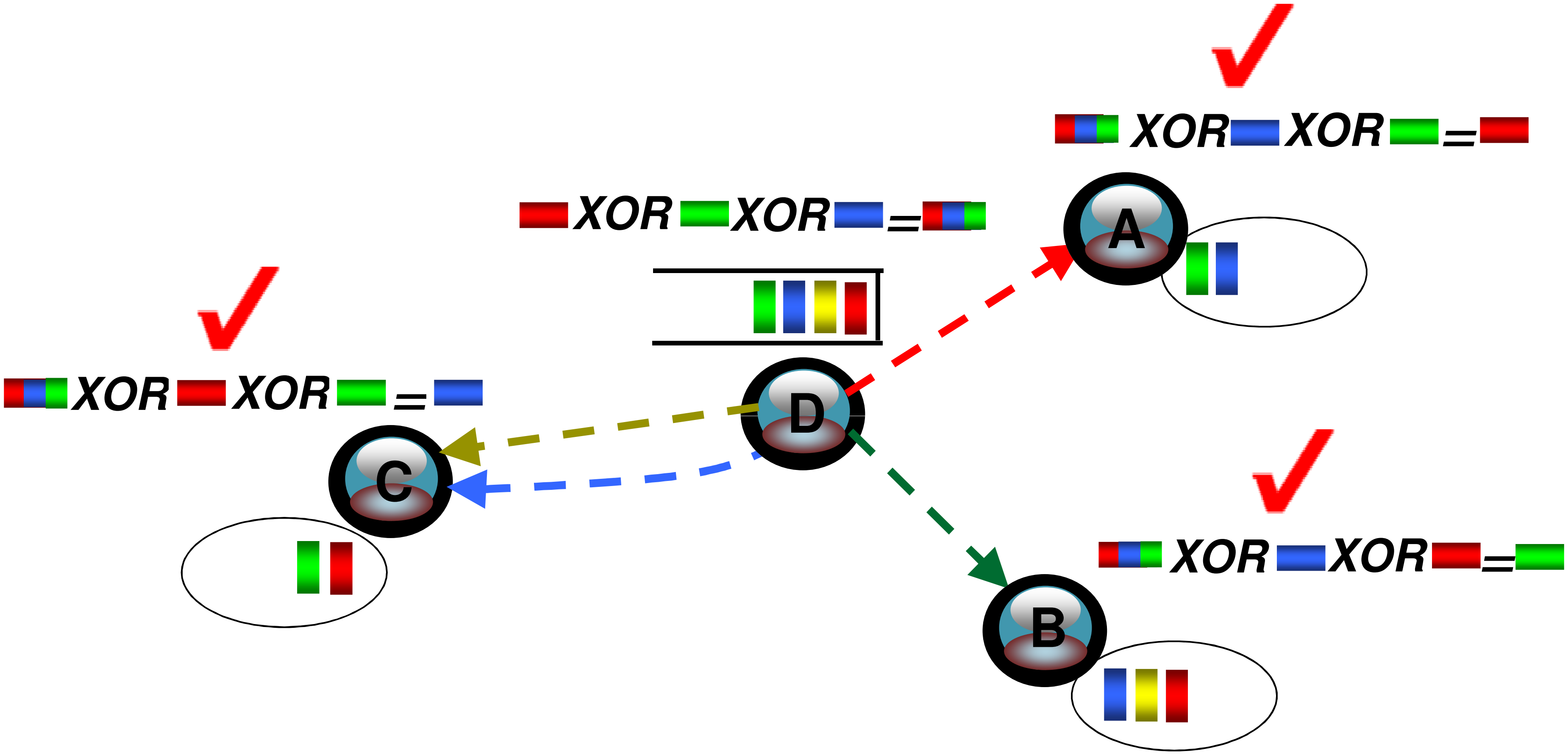
Which Packets to Code Together?



Good Coding

Two neighbors benefit from one transmission!

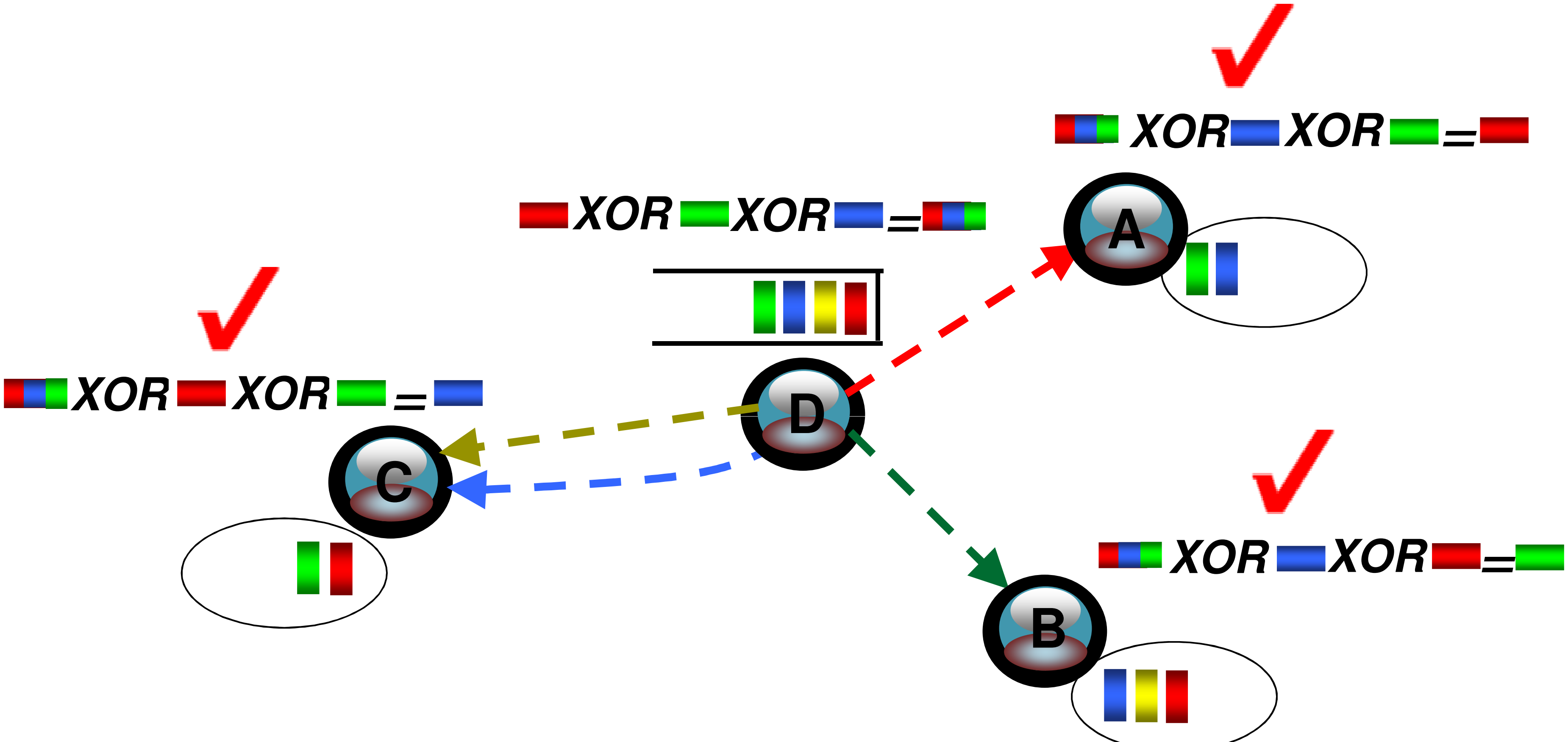
Which Packets to Code Together?



Best Coding

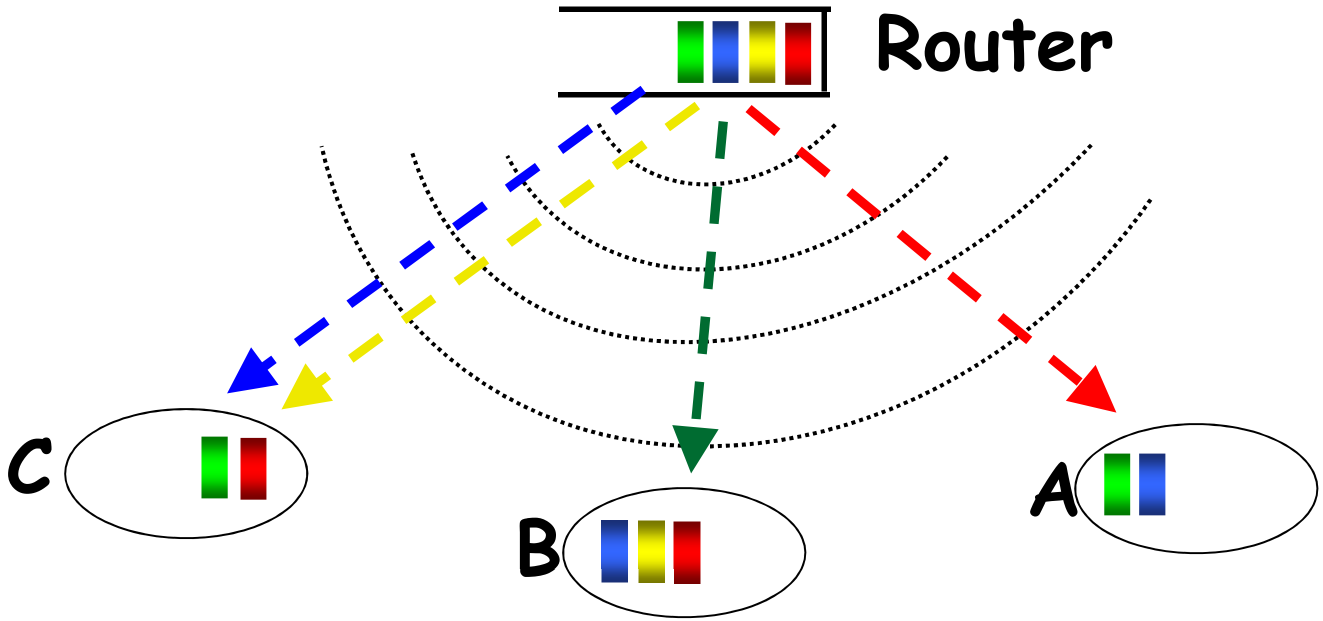
Three neighbors benefit from one transmission!

Which Packets to Code Together?



XOR n packets together iff the next hop of each packet already has the other ***n-1*** packets apart from the one it wants

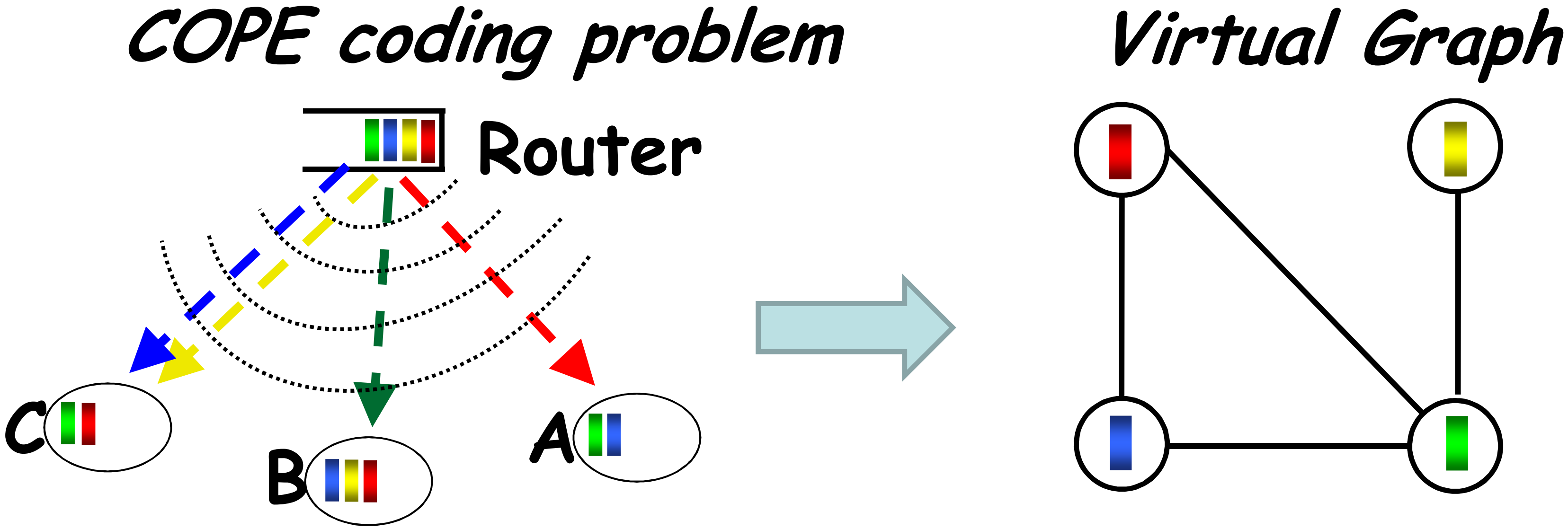
Problem Formalization



- Router has *n* packets, each node has a different subset of these packets and needs a specific subset
- Minimize number of transmissions subject to coding constraints

Solution Intuition

- Virtual graph: Each packet is a vertex, edge if corresponding packets can be coded together



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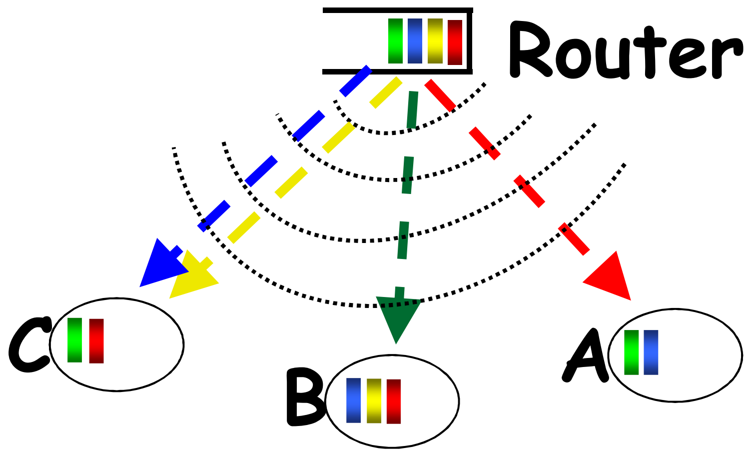
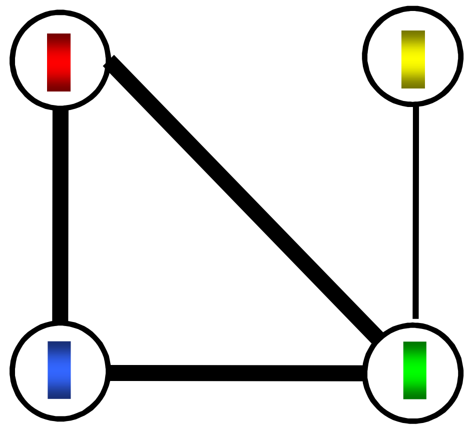
COPE coding problem



Virtual Graph

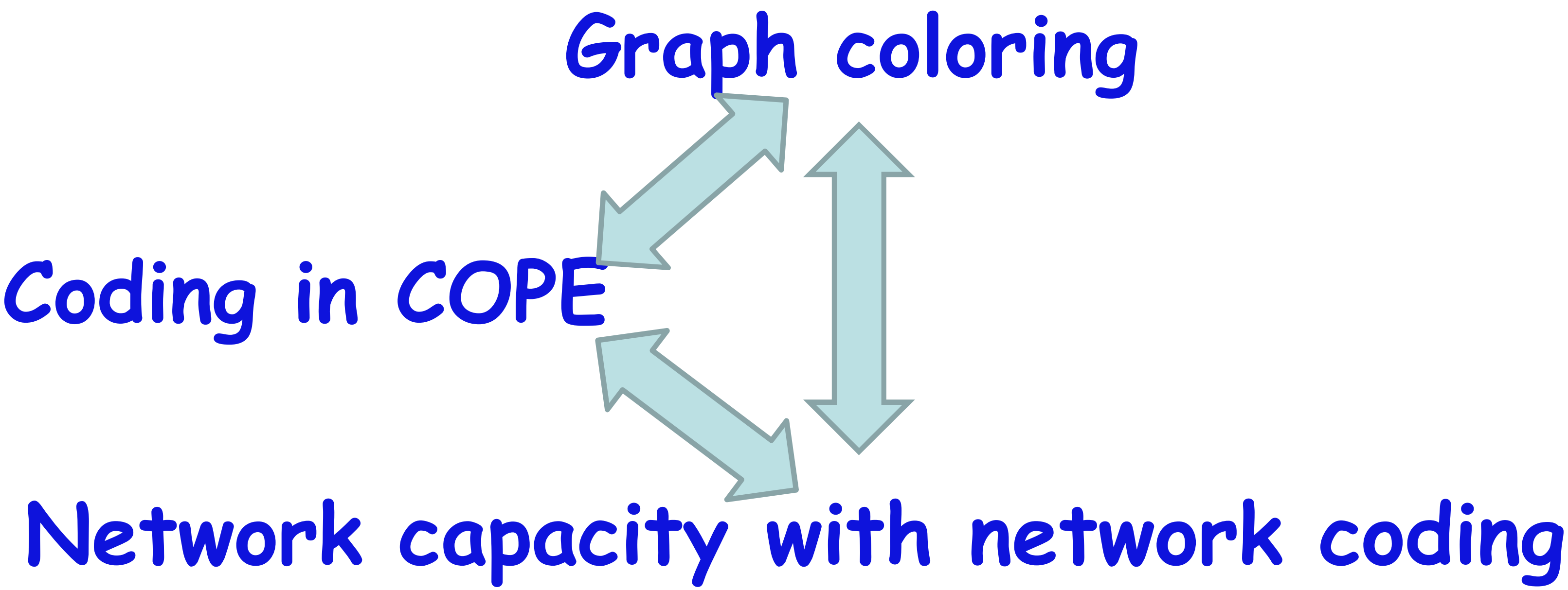


Coded packet → **Clique** in virtual graph

Minimizing no. of transmissions
→ Clique partitioning in virtual graph

Solution Intuition

- COPE's coding problem → Clique partitioning in virtual graph
- Clique partitioning → Graph coloring in complementary graph
- NP hard, heuristic solution in paper



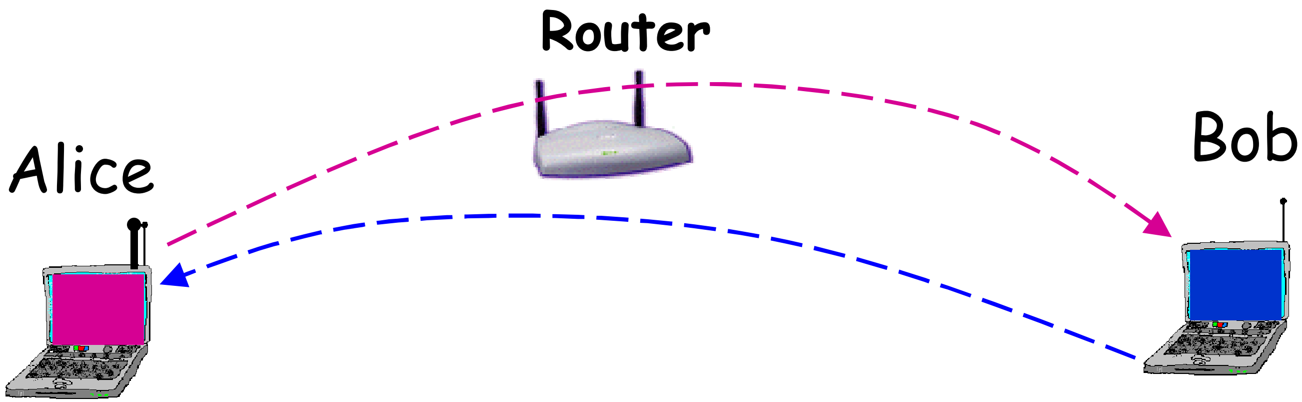
COPE's Characteristics

- COPE is a forwarding mechanism
  - It sits transparently between IP and MAC
  - Routing is unmodified (i.e., shortest path)
- Opportunistic → Code packets if possible, if not forward without coding
- Does not delay packets

Performance

COPE is implemented in Linux

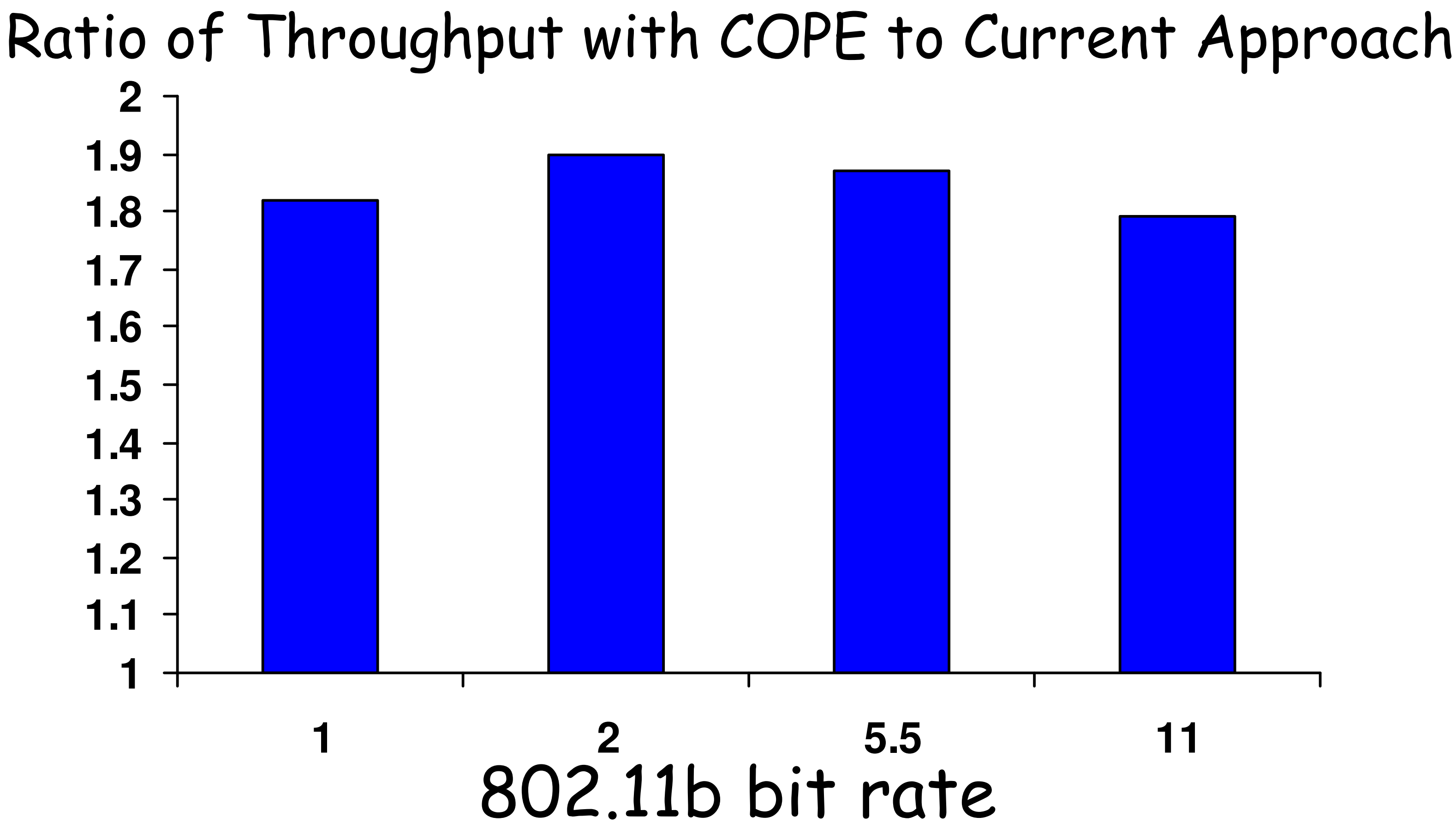
Alice-and-Bob Experiment



3 transmissions instead of 4

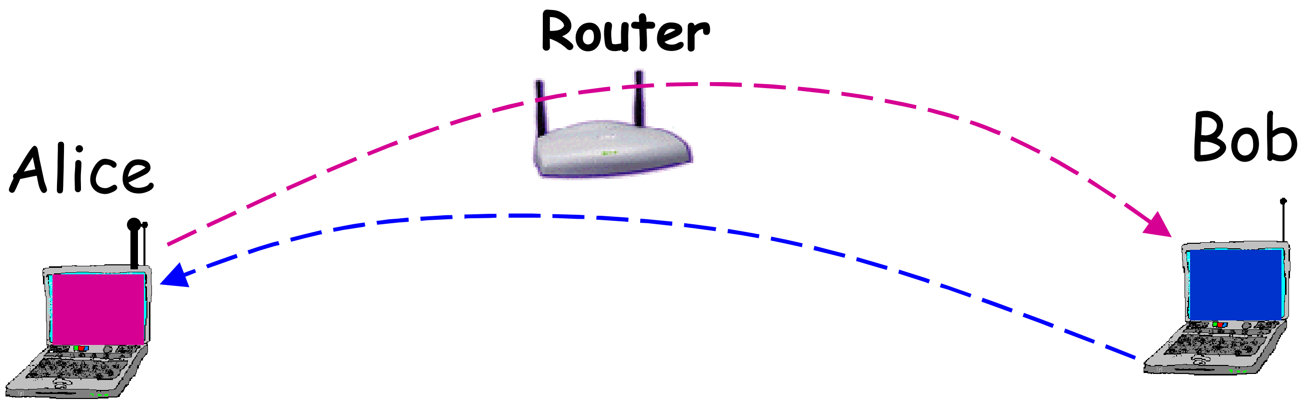
→ Throughput gain is 4/3 = 1.33

Results of the Alice-and-Bob



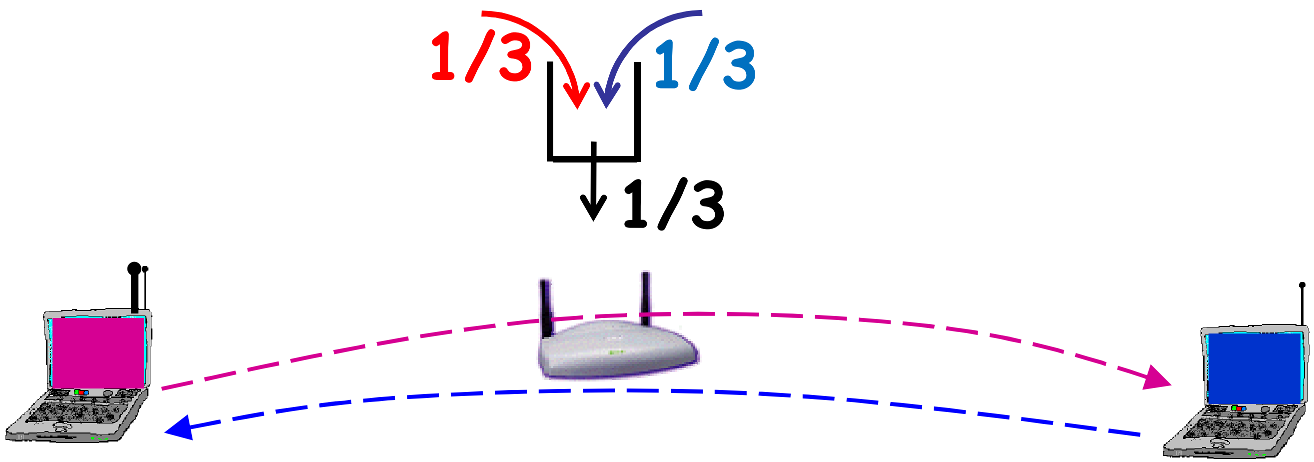
COPE almost doubles the throughput

Why More Gain than 1.33?



COPE alleviates the mismatch between MAC's capacity allocation and the congestion at a node

Why More Than 1.33?



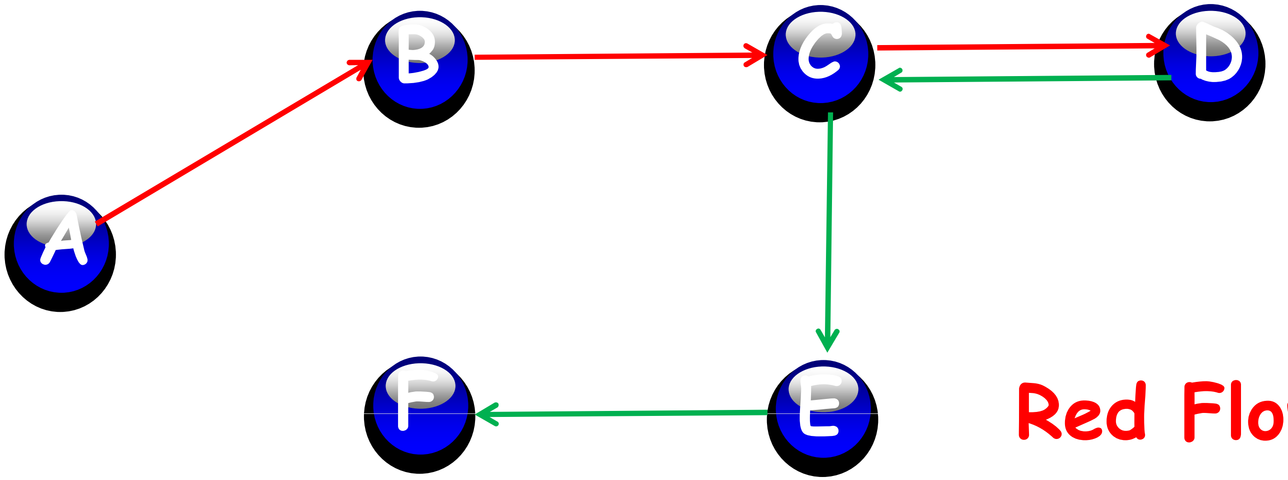
COPE alleviates the mismatch between MAC's allocation and the congestion at a node

Coding Gain	Coding+MAC Gain
Reduction in #Transmissions	Improvement of Draining Rate at Bottlenecks
In Alice-Bob scenario, Coding Gain is 4/3 = 1.33	In Alice-Bob scenario, Coding+MAC Gain is 2
Nodes not backlogged	Nodes backlogged

Can show that

Coding gain is bounded by 2	Coding+MAC gain can be infinite

Extension: Coding-Aware Routing [SRB07]



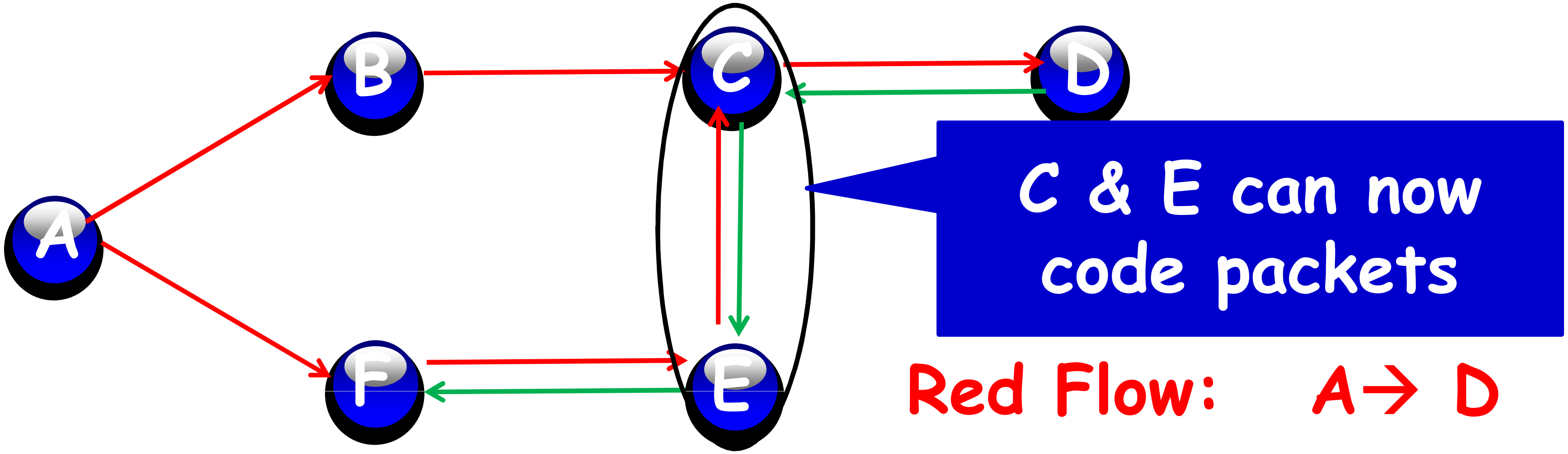
Red Flow: A→ D

Green Flow: D→ F

6 transmissions to send two packets

No coding opportunities since flows take different routes

Extension: Coding-Aware Routing [SRB07]



Red Flow: A→ D

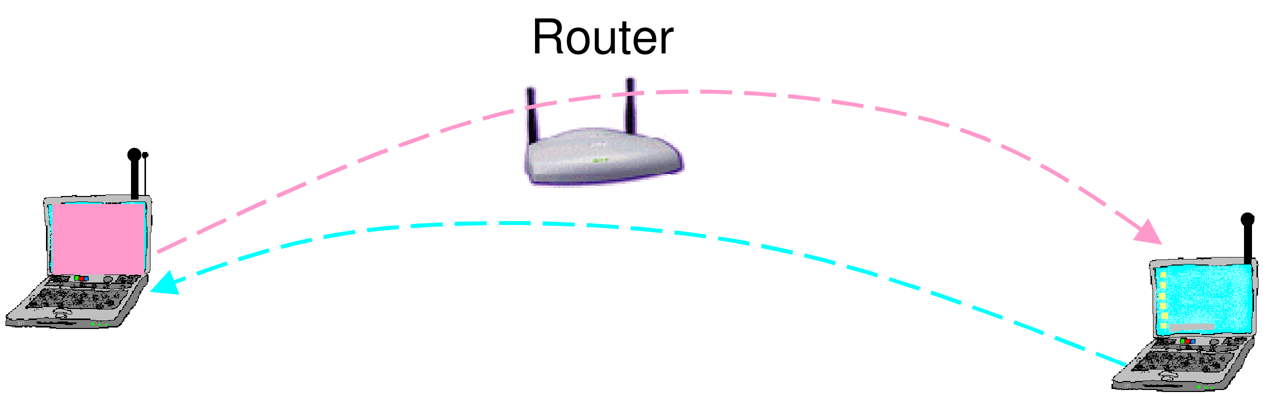
Green Flow: D→ F

Pick routes that enable coding

5 transmissions instead of 6

Modified routing that maximizes throughput given coding

Can We Do Better?



Current approach requires 4 time slots

Network coding requires 3 time slots

How about 2 time slots?

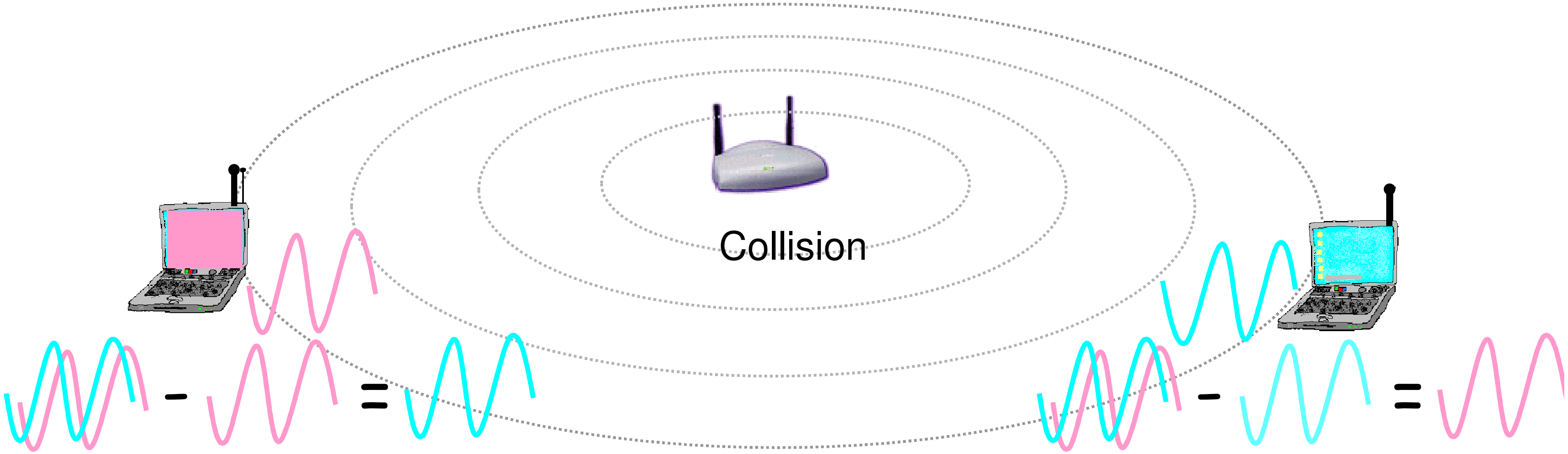
Instead of router mixing packets…

Exploit that the wireless channel naturally mixes signals when packets interfere!

Analog Network Coding

Analog Network Coding

Analog Network Coding



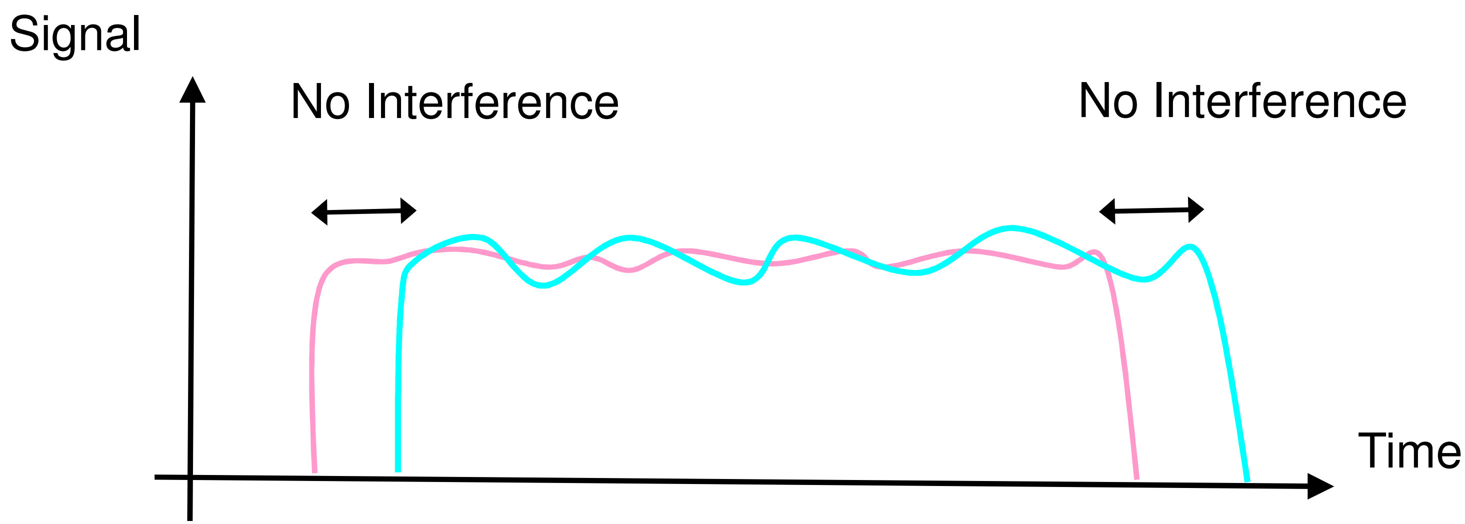
- Alice and Bob send simultaneously
- Router receives the sum of the two signals (plus time and phase shifts)

2 Time Slots → Even Higher Throughput

Challenges

- Interfered signal is not really the sum
  - Channel distort signal
  - Two signals are never synchronized
  - It is not $A(t) + B(t)$ but $f_1(\ A(t)\ ) + f_2(\ B(t-T)\ )$

Solution: The Bliss of Asynchrony



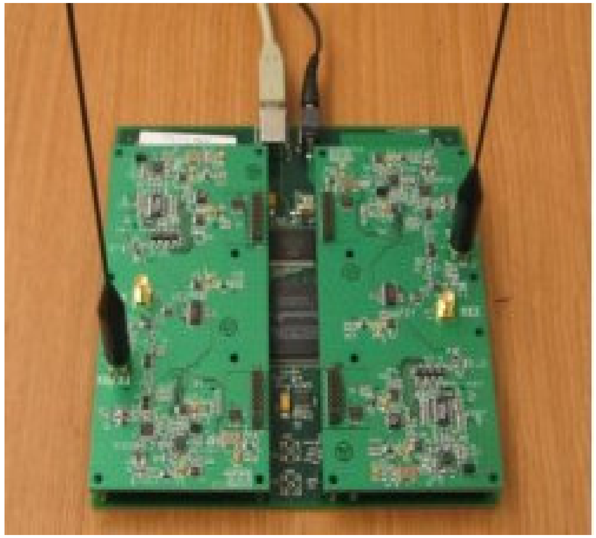
- Alice uses non-interfering bits from her signal to estimate her channel
- Alice compensates for her interfering signal

We exploit the lack of synchronization!

Design & Implementation

- ANC works with any modulation scheme
  - Accurate channel, frequency & sampling offset estimation
  - Iterative algorithms to deal with interference
- Linear decoding complexity
- Implemented in software radios

(GMSK, BPSK, QPSK, …)

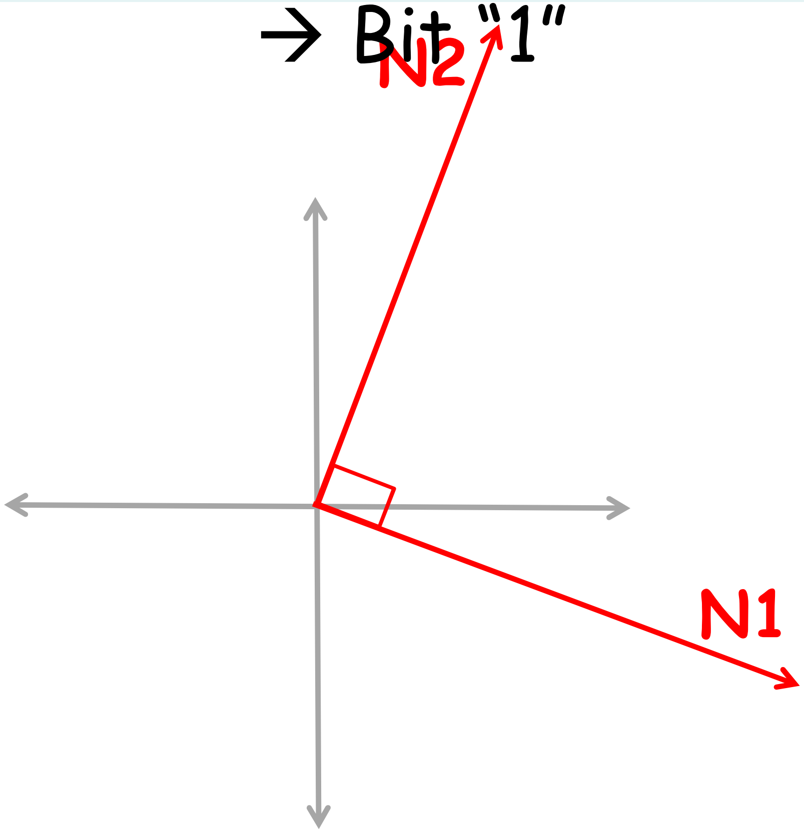


Primer on Modulation

- Digital version of wireless signals → Complex samples

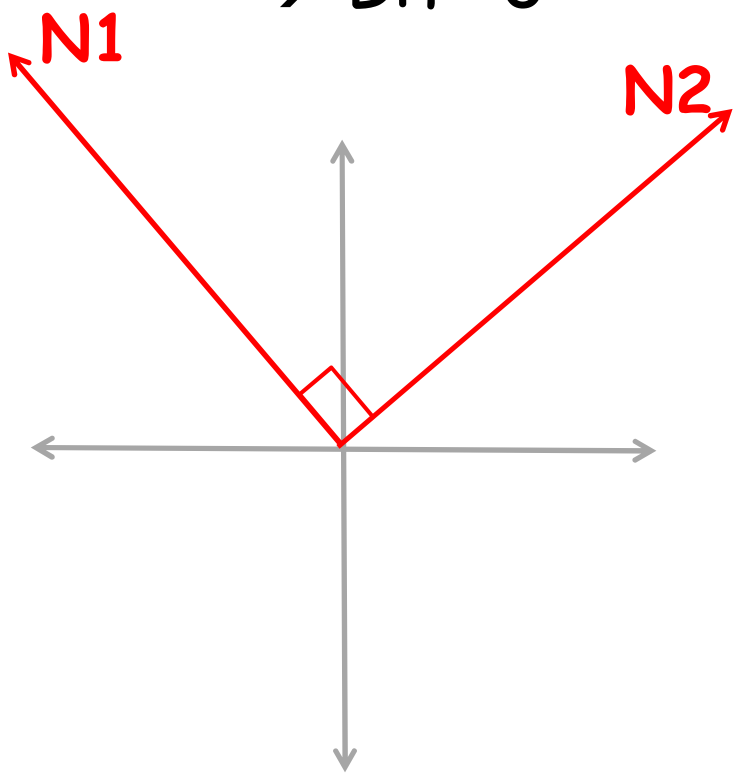
N2 leads N1 by 90 degrees

→ Bit "1"



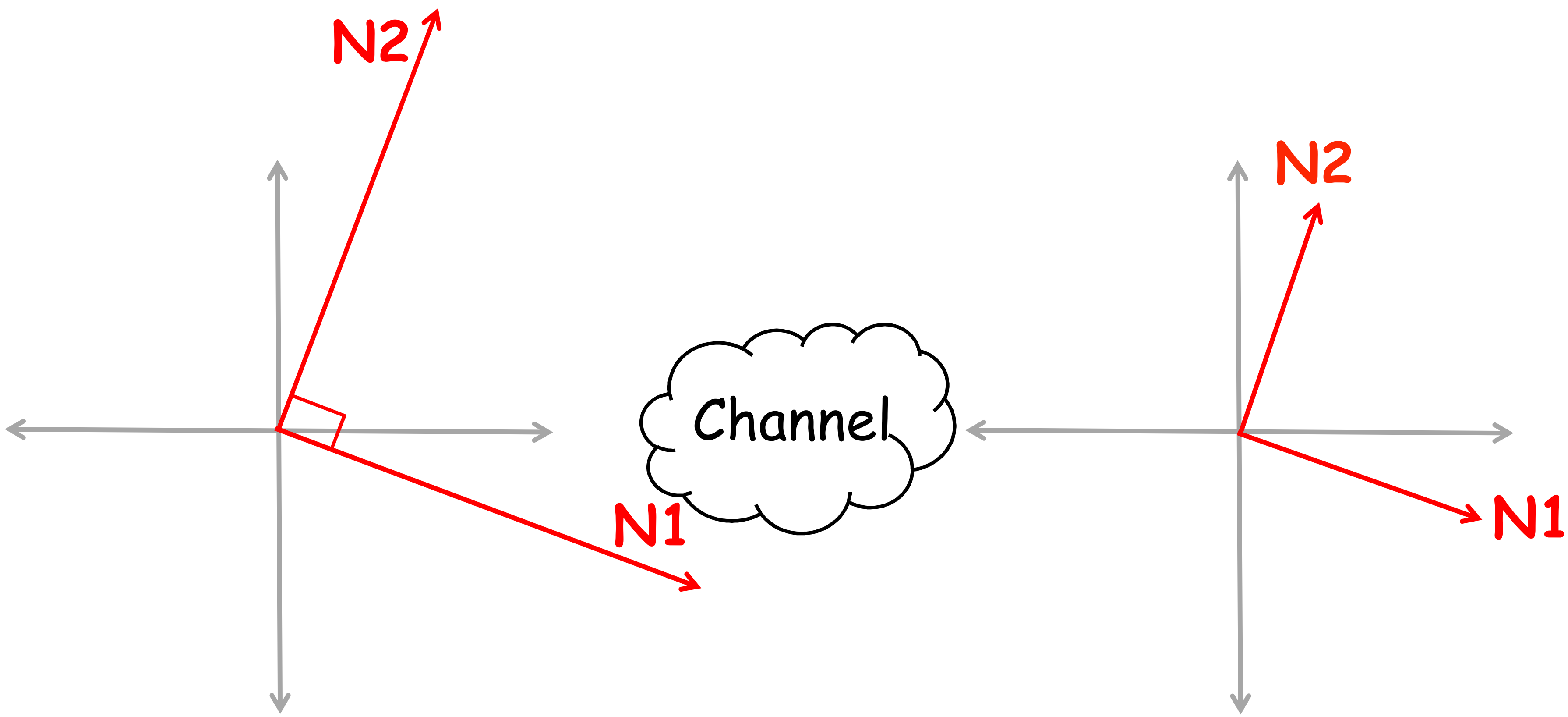
N2 lags N1 by 90 degrees

→ Bit "0"



Primer on Channel Effects

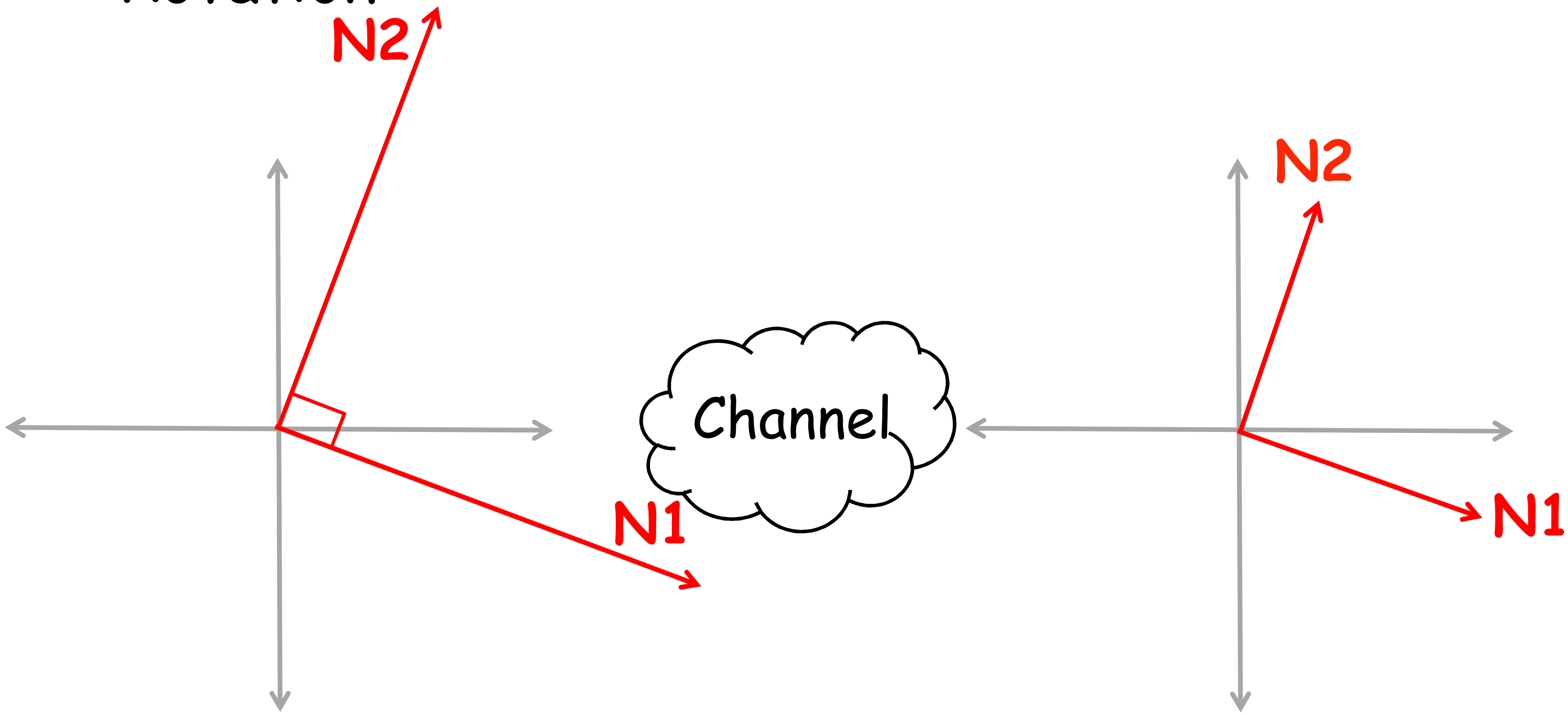
- Attenuation



N2 and N1 are attenuated by the same amount

Primer on Channel Effects

- Attenuation
- Rotation



Primer on Channel Effects

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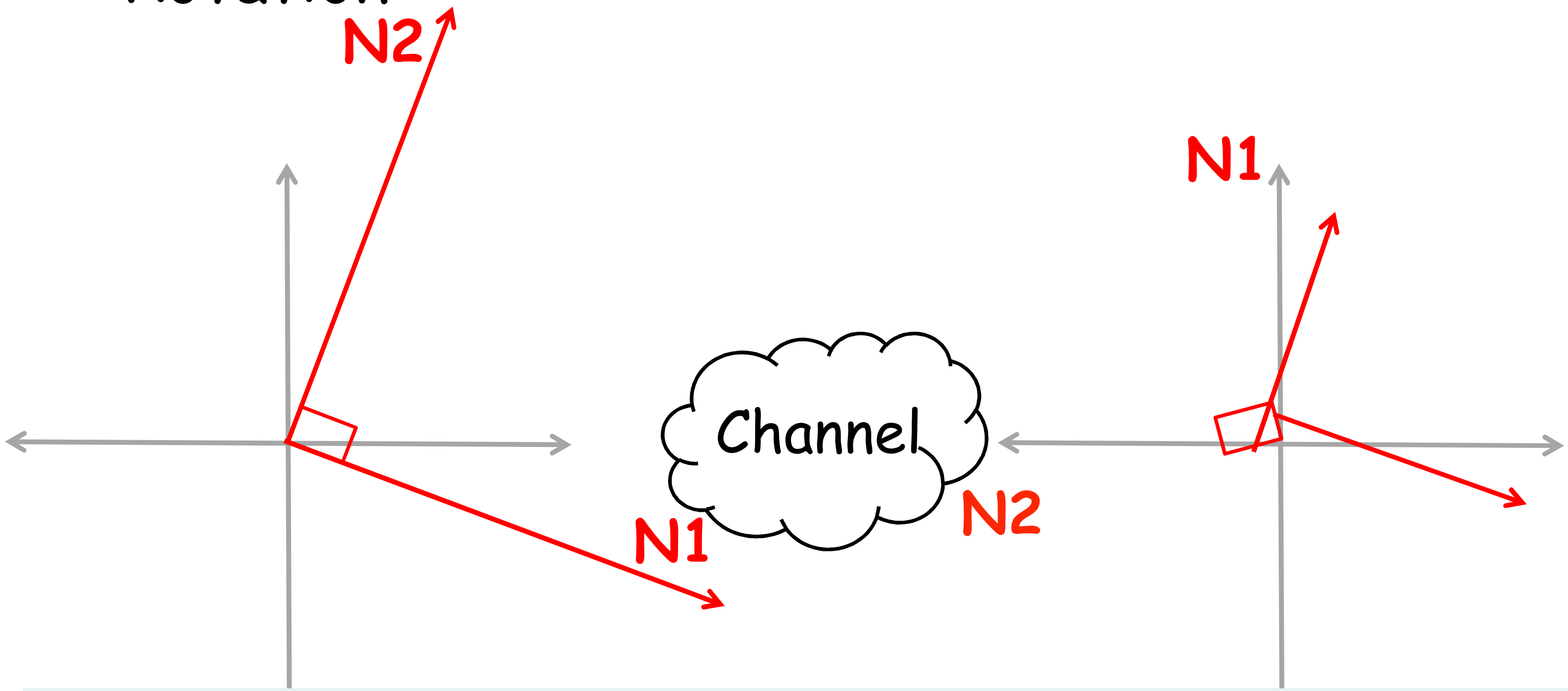
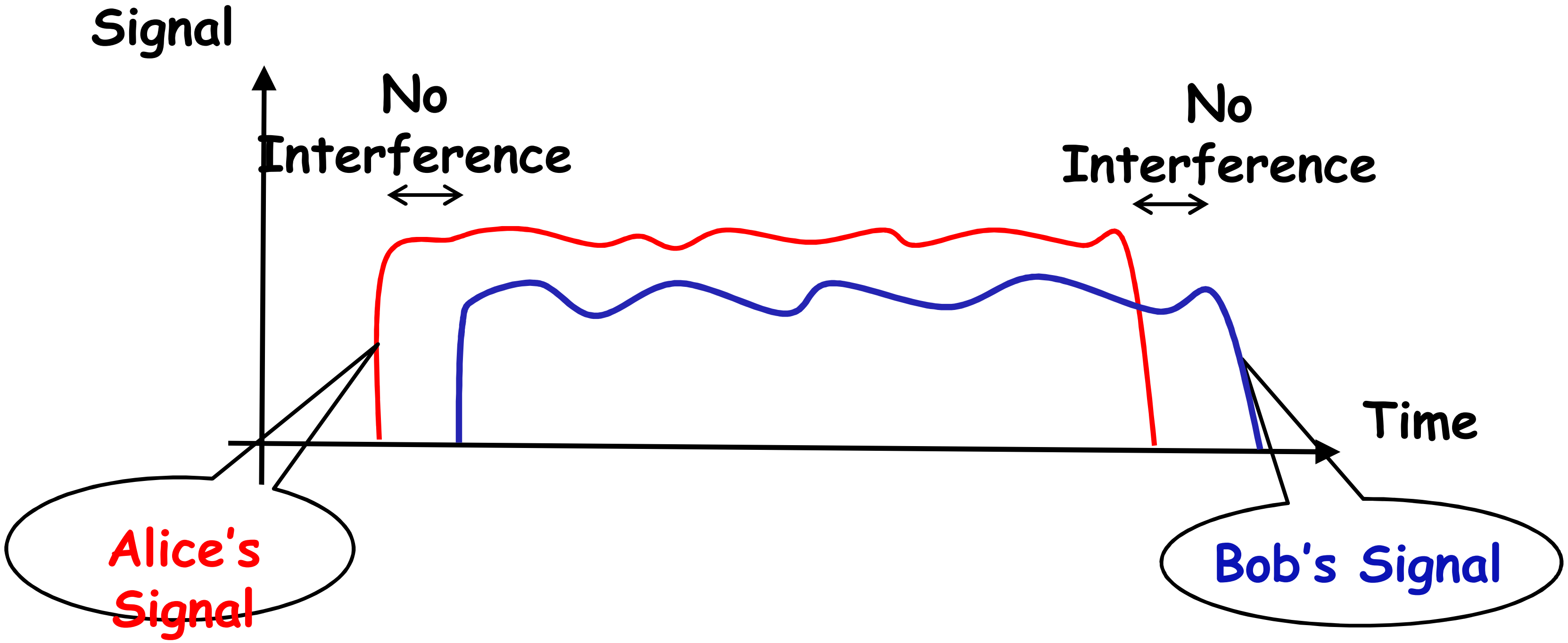


To decode, compute angle between received complex numbers **Angle (N2, N1)** = 90 degrees→ Bit "1" was transmitted

So, How Does Alice Decode?



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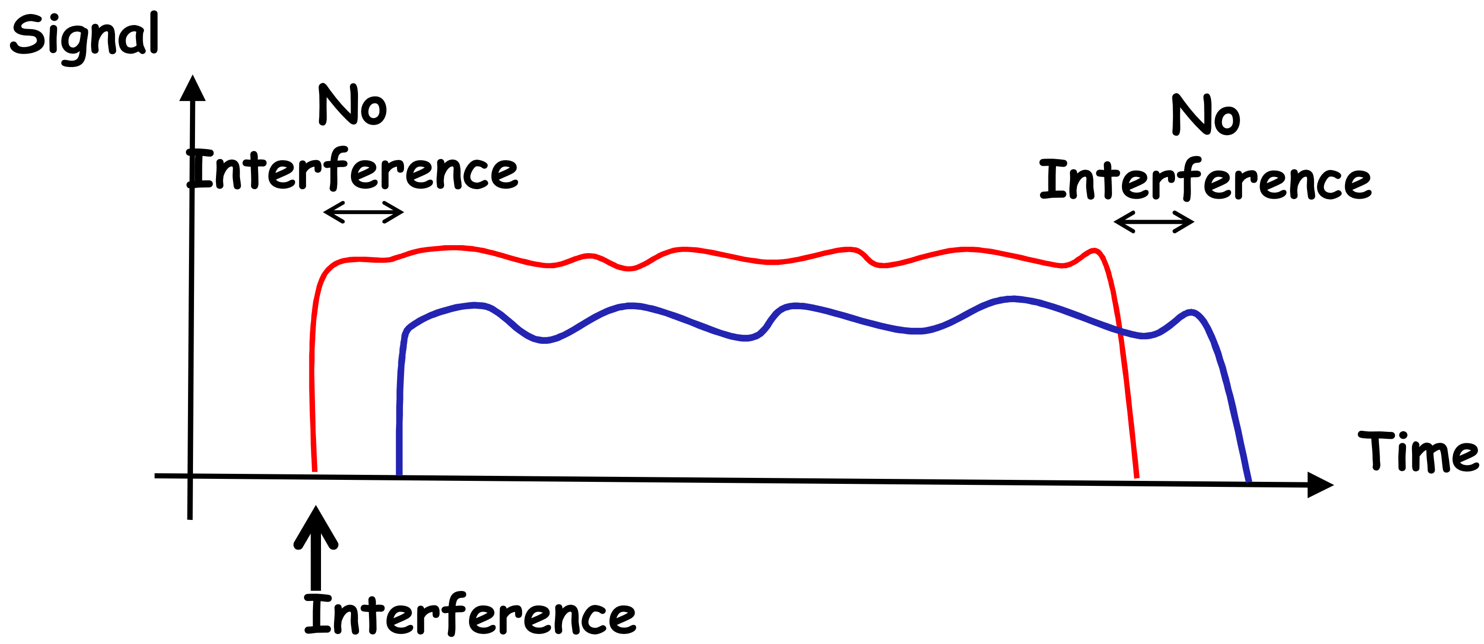
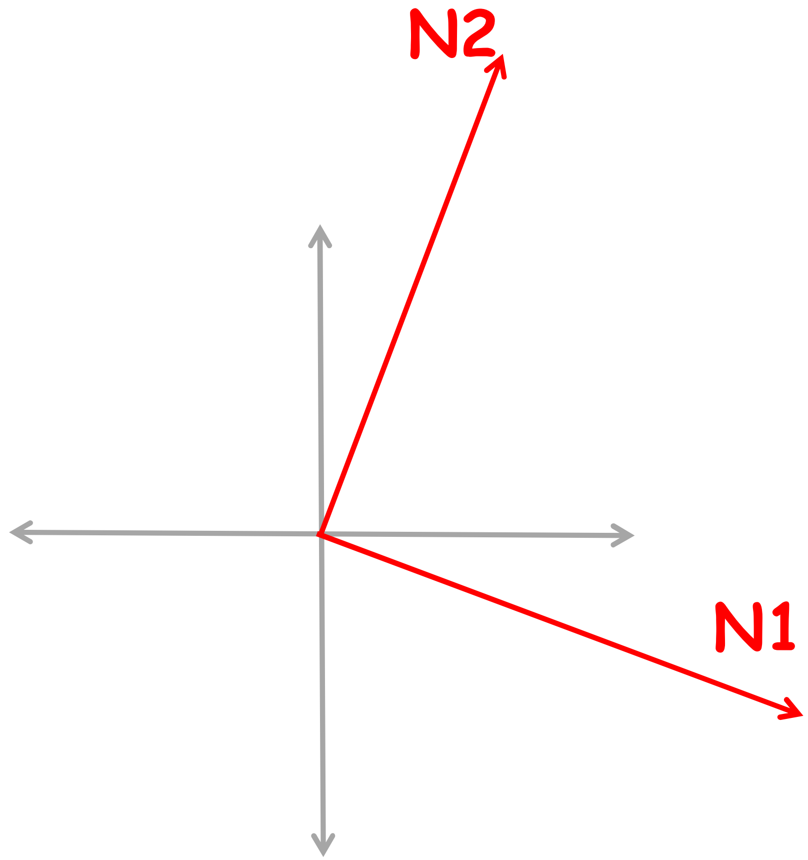
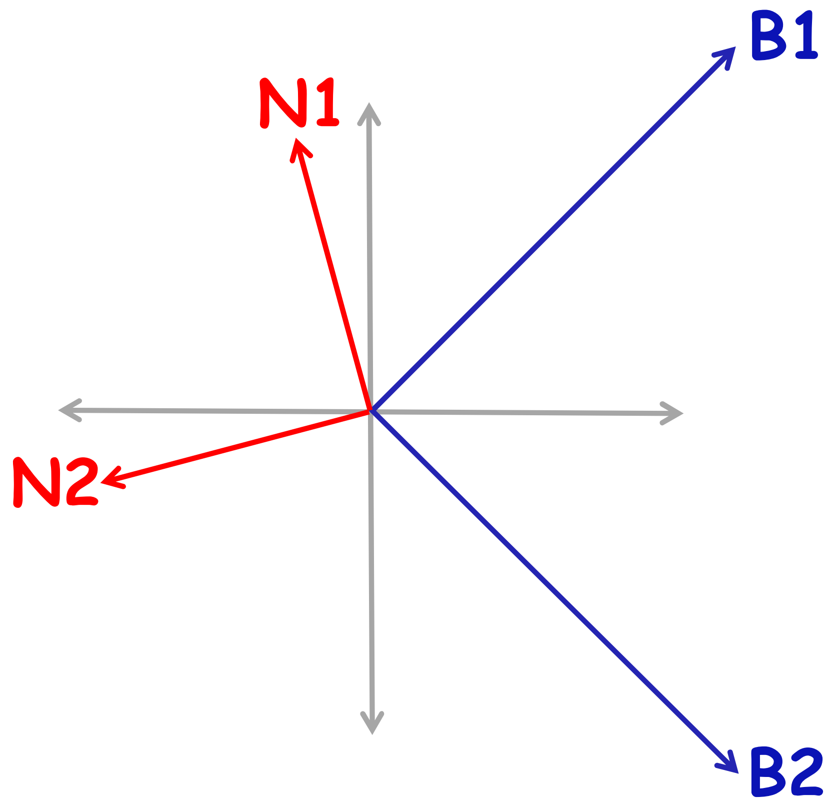


- Small uninterfered part at the start
- Decodes uninterfered part via standard GMSK demo
- Once interference starts, Alice changes decoding algorithm

- What did Alice send?



- What did Alice send?
- What did Bob send?



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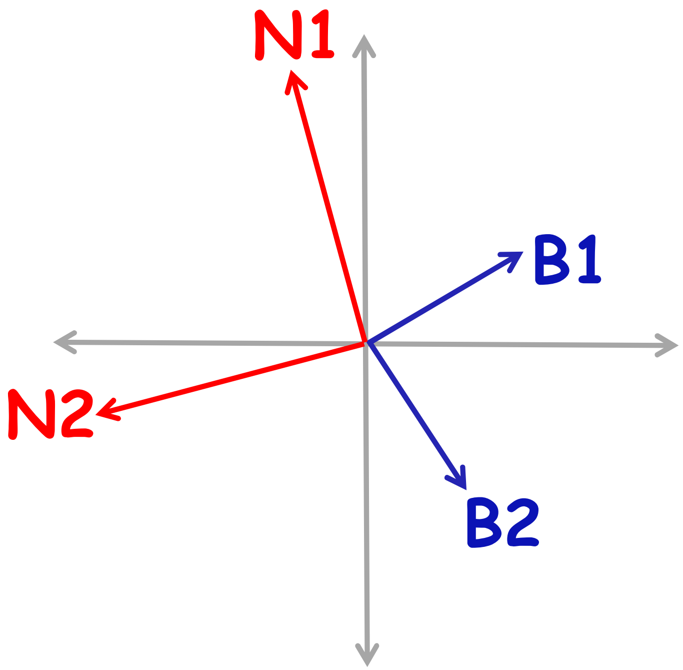
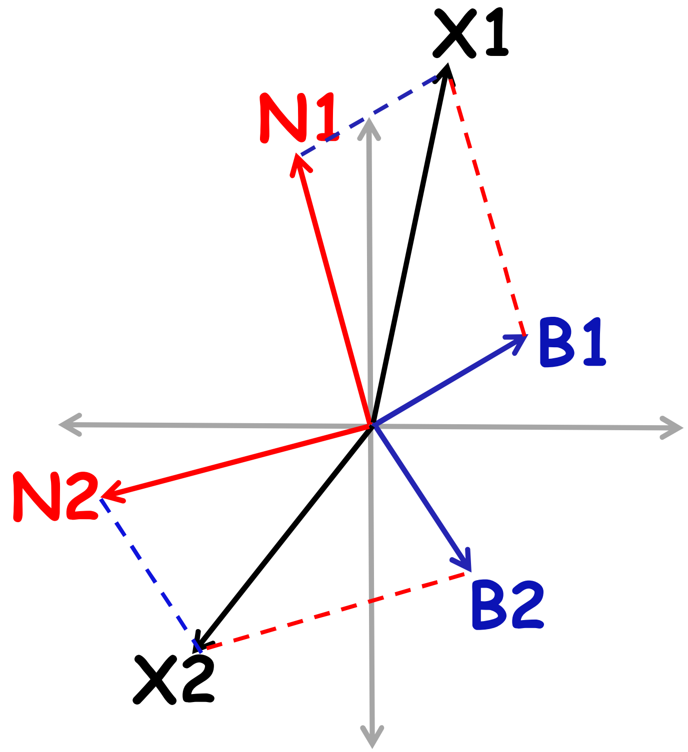
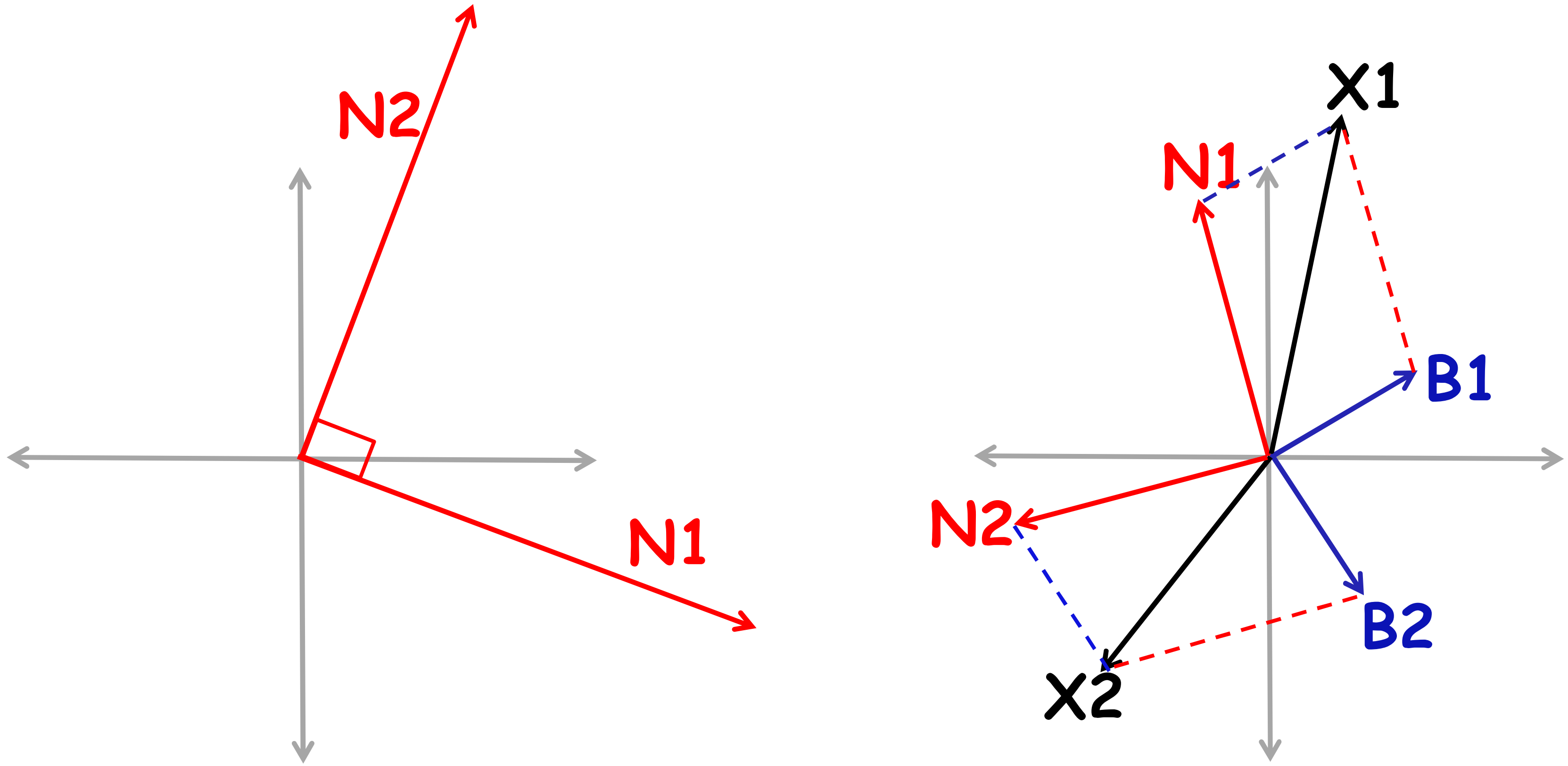


- What is Interference → Complex addition



- What does Alice know?



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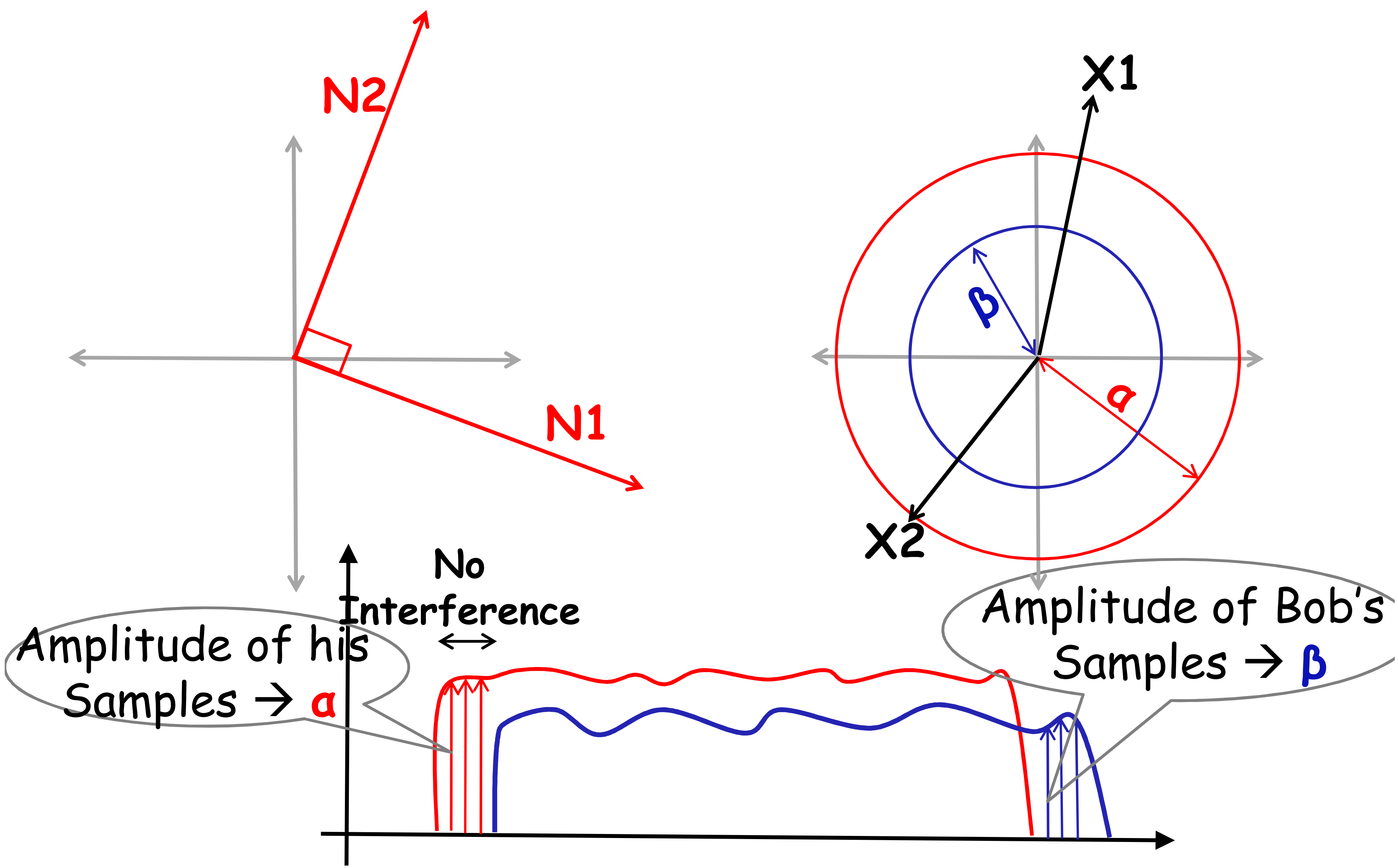


Solutions for interfered complex sample

Interfered complex no: $X = \alpha e^{j\theta} + \beta e^{j\varphi}$

Lemma:

"If X is an interfered complex number satisfying the above equation, then the pair $[\theta, \varphi]$ takes one of the following two values:

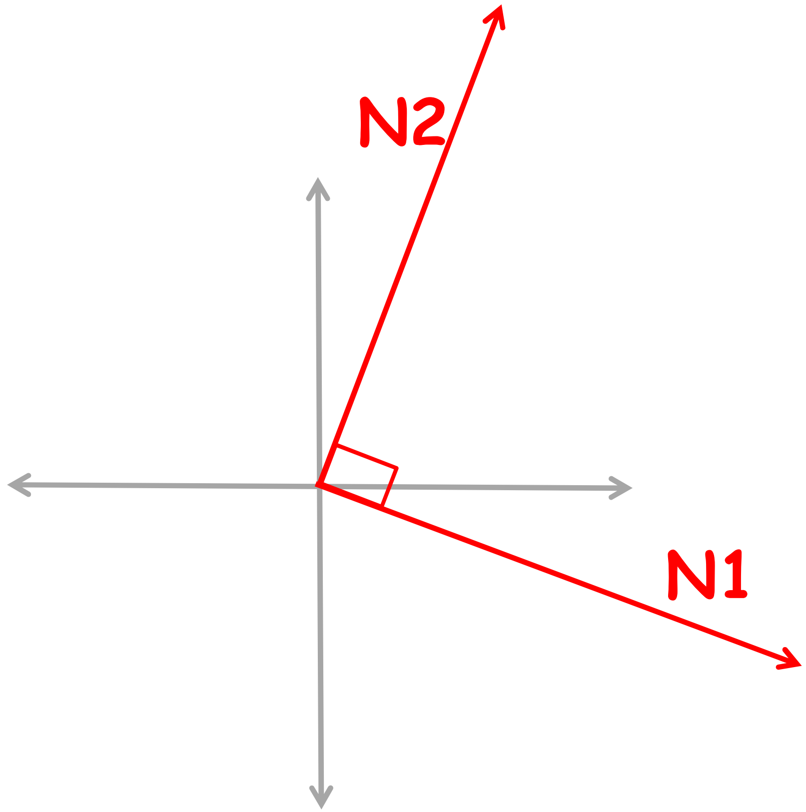
$$\theta = \arg(X(\alpha + \beta D \pm j\beta\sqrt{1-D^2}))$$

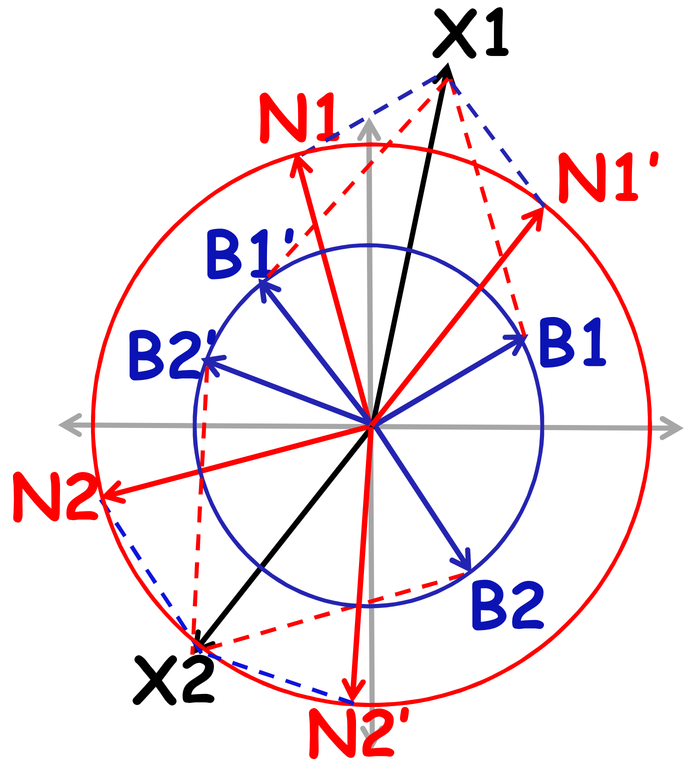
$$\varphi = \arg(X(\beta + \alpha D \mp j\alpha\sqrt{1-D^2}))$$

where

$$D = \frac{|X|^2 - \alpha^2 - \beta^2}{2\alpha\beta}$$

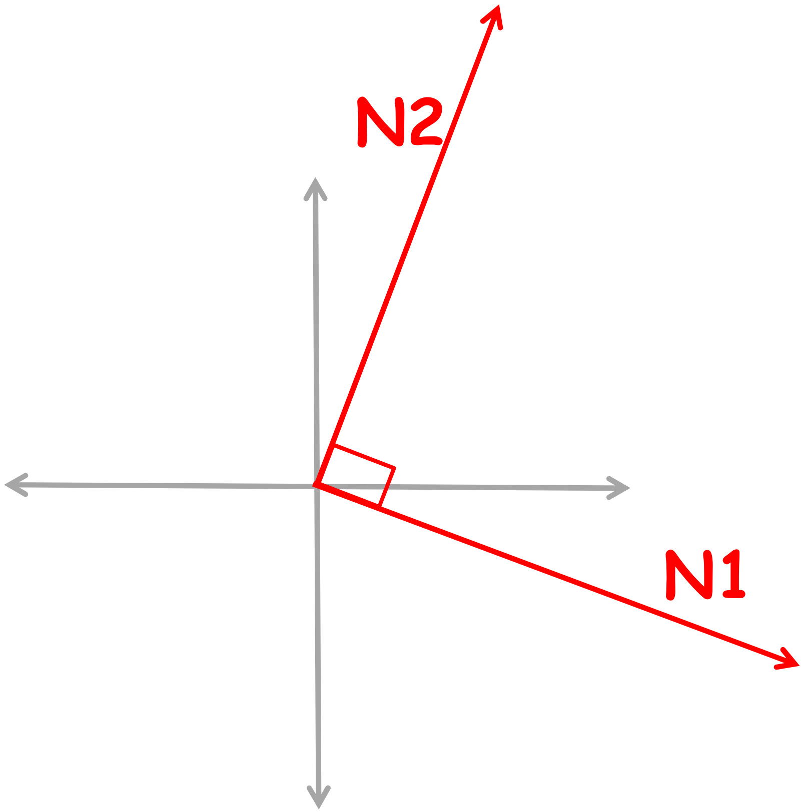
- What does Alice know?

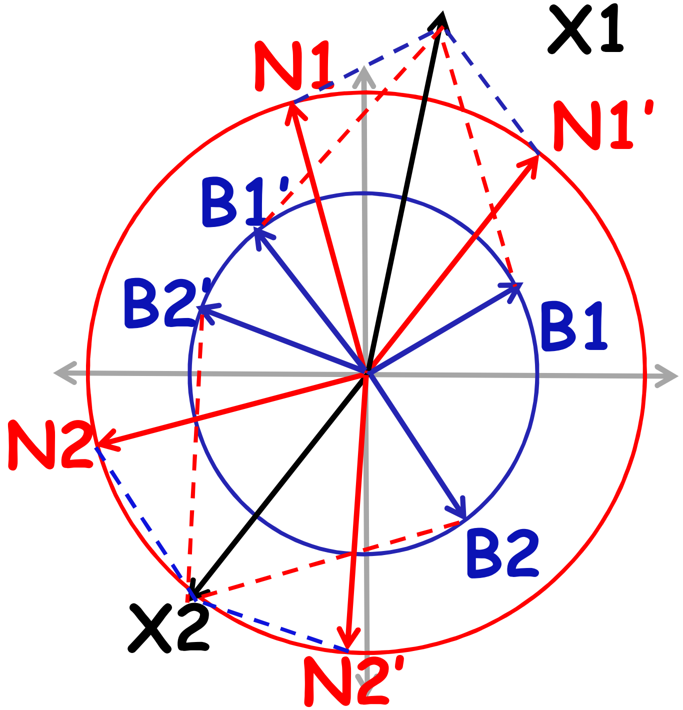




Two solutions for each interfered complex sample!

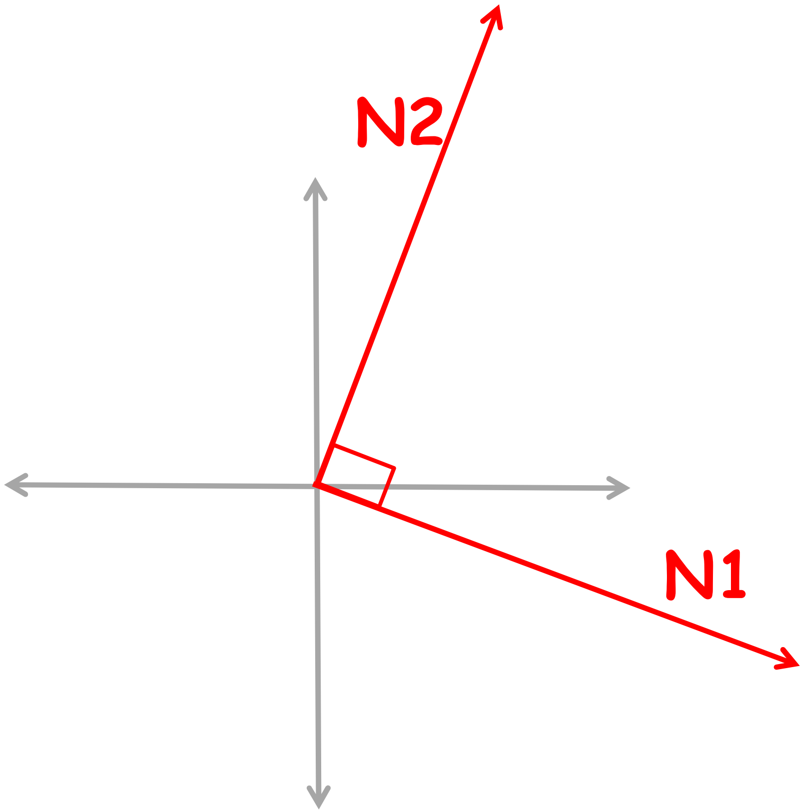
- What does Alice know?

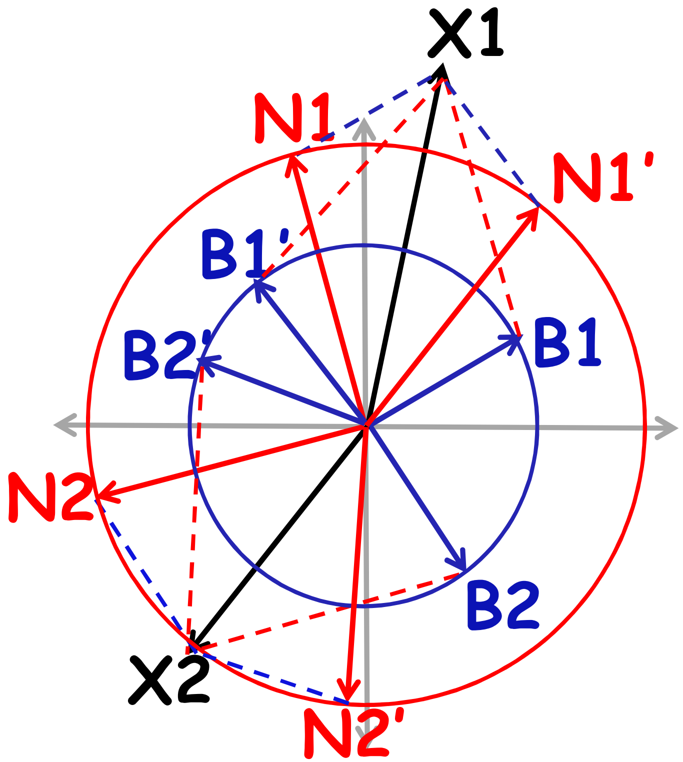




Two solutions for each interfered complex sample!

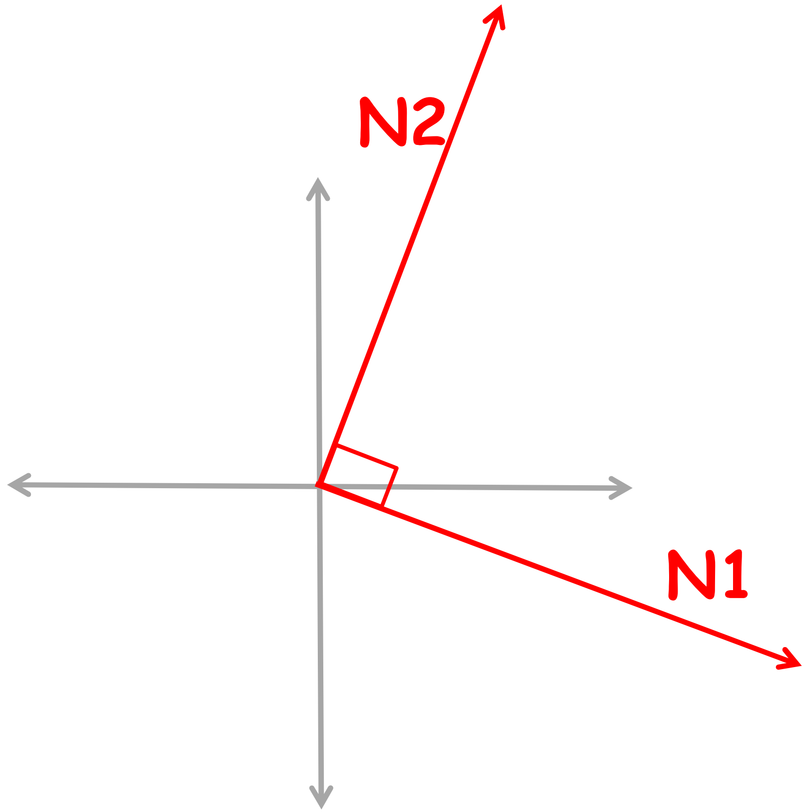
- What does Alice know?

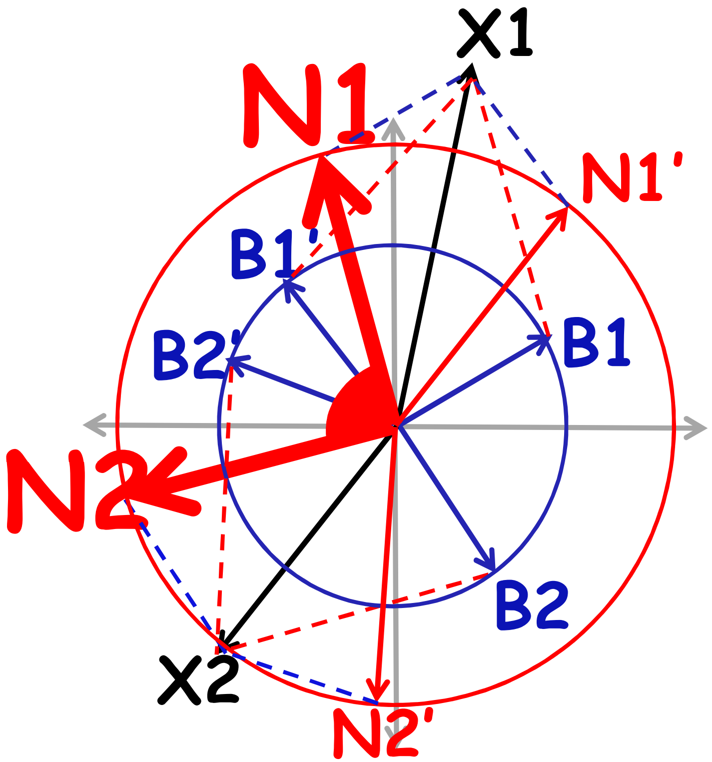




Four possible angles!

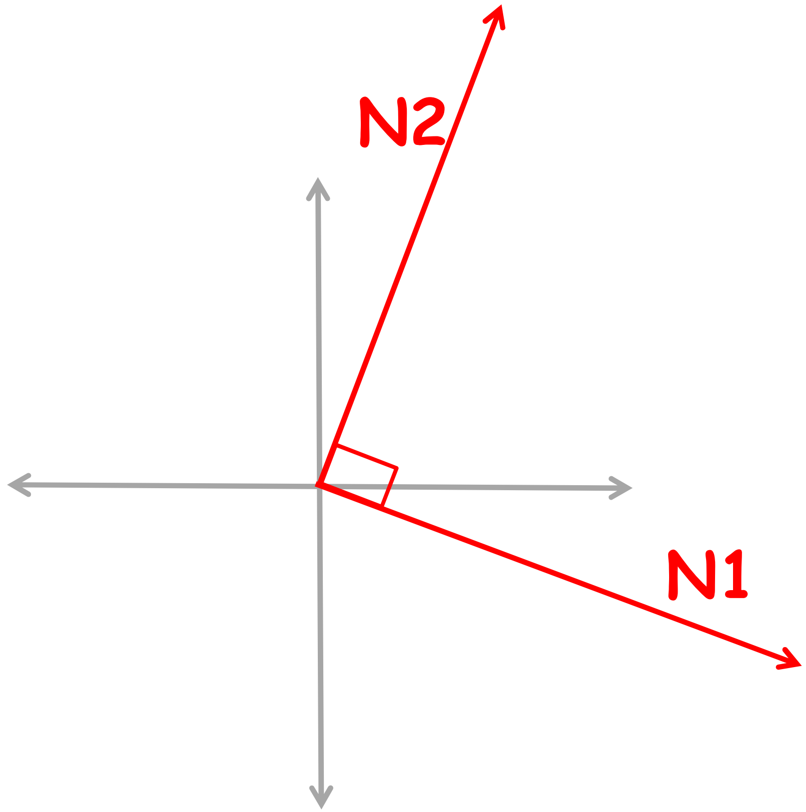
- What does Alice know?

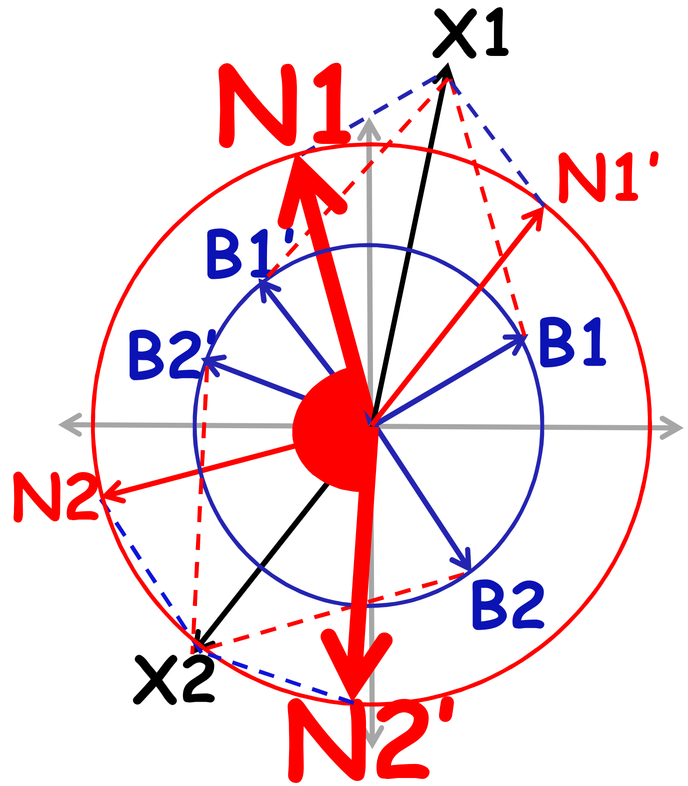




Four possible angles!

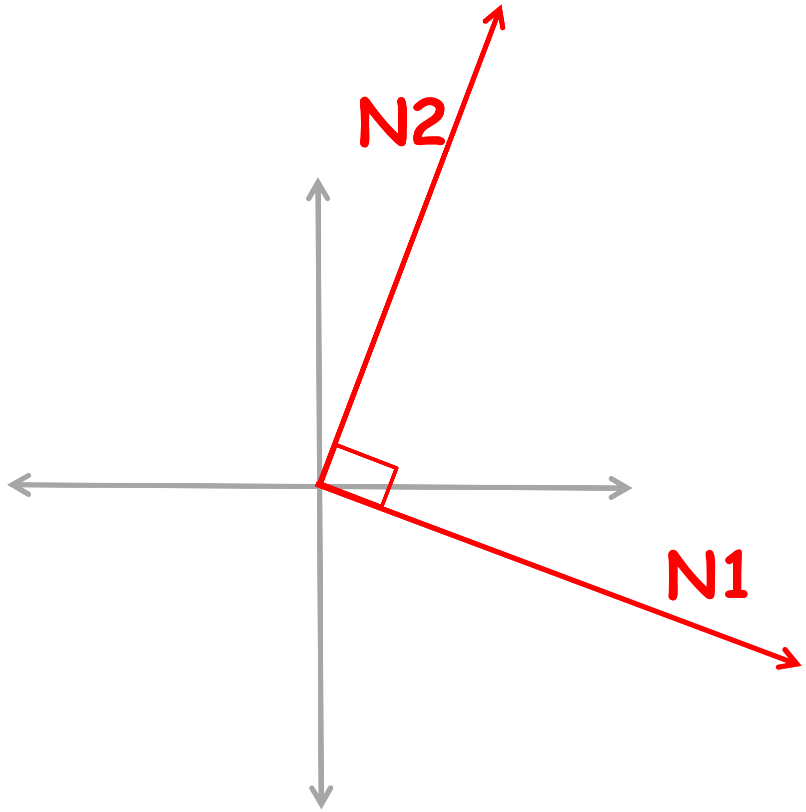
- What does Alice know?

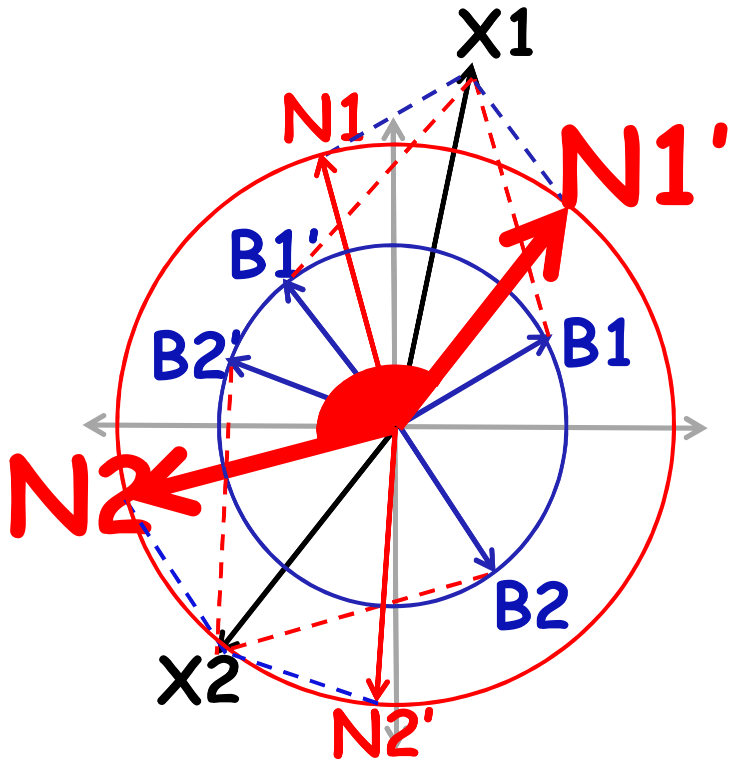




Four possible angles!

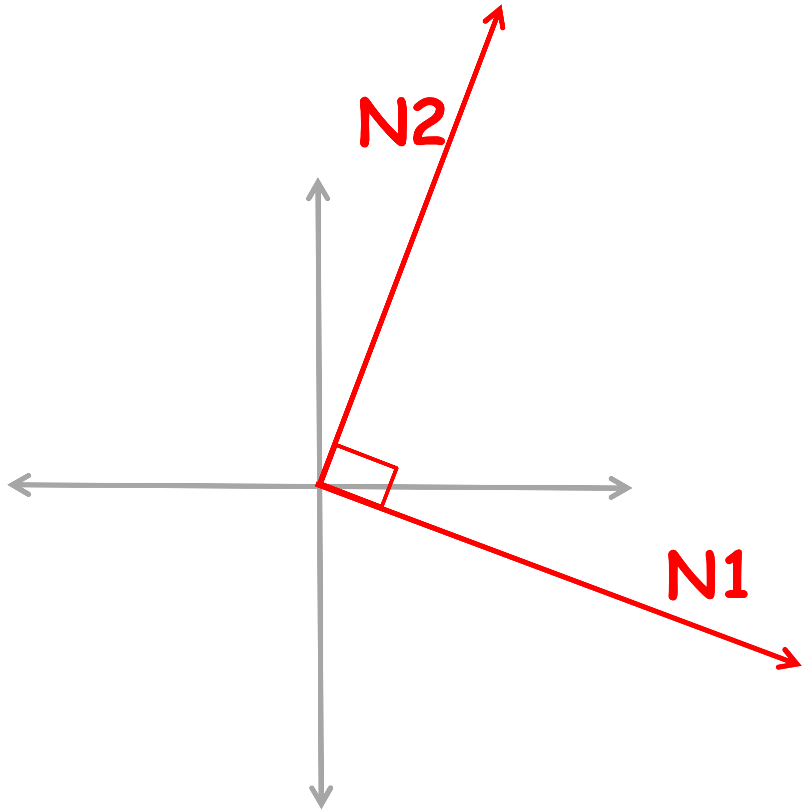
- What does Alice know?

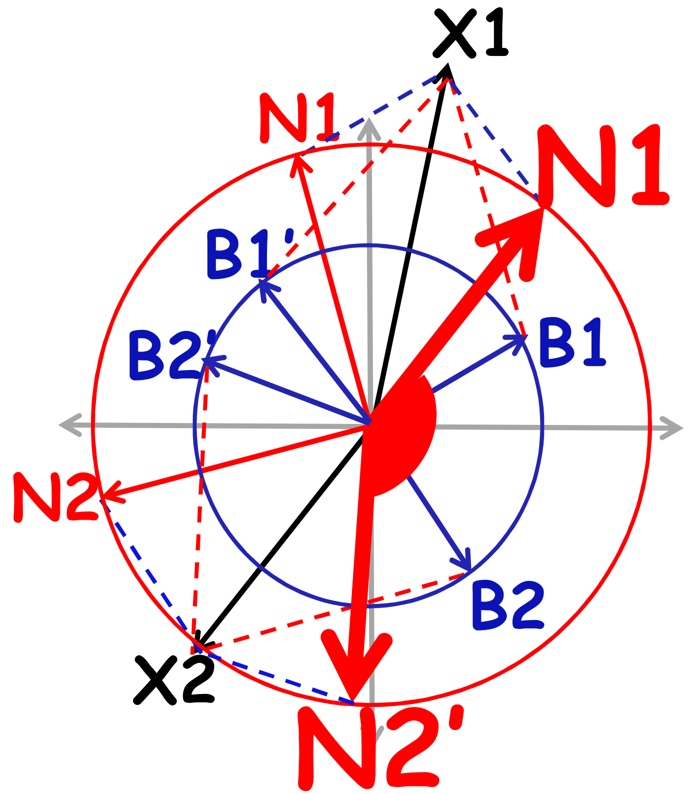




Four possible angles!

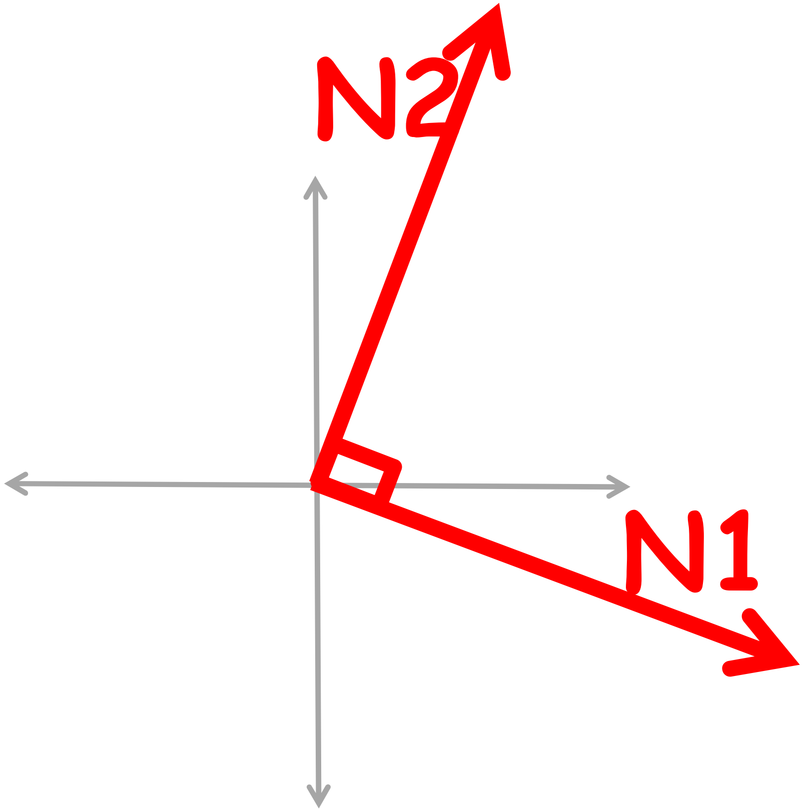
- What does Alice know?

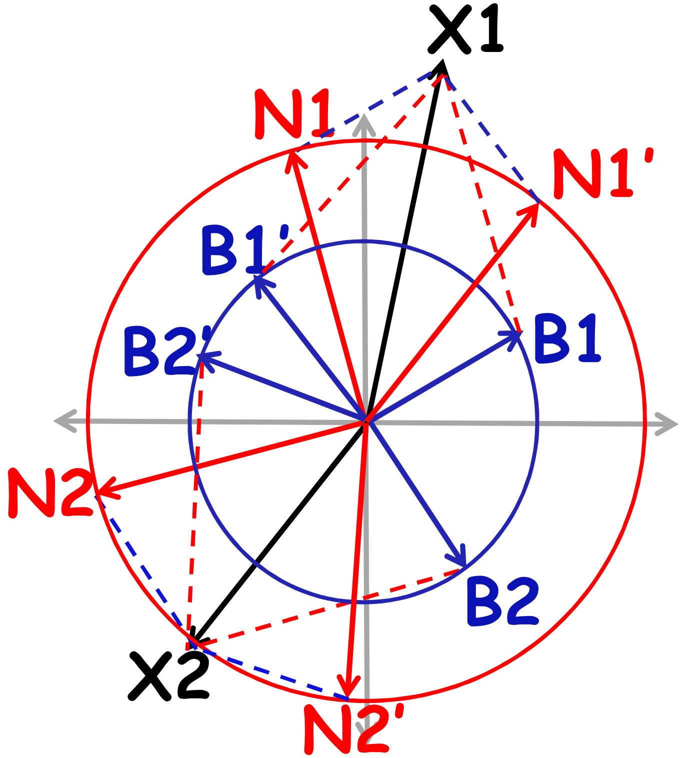




Four possible angles!

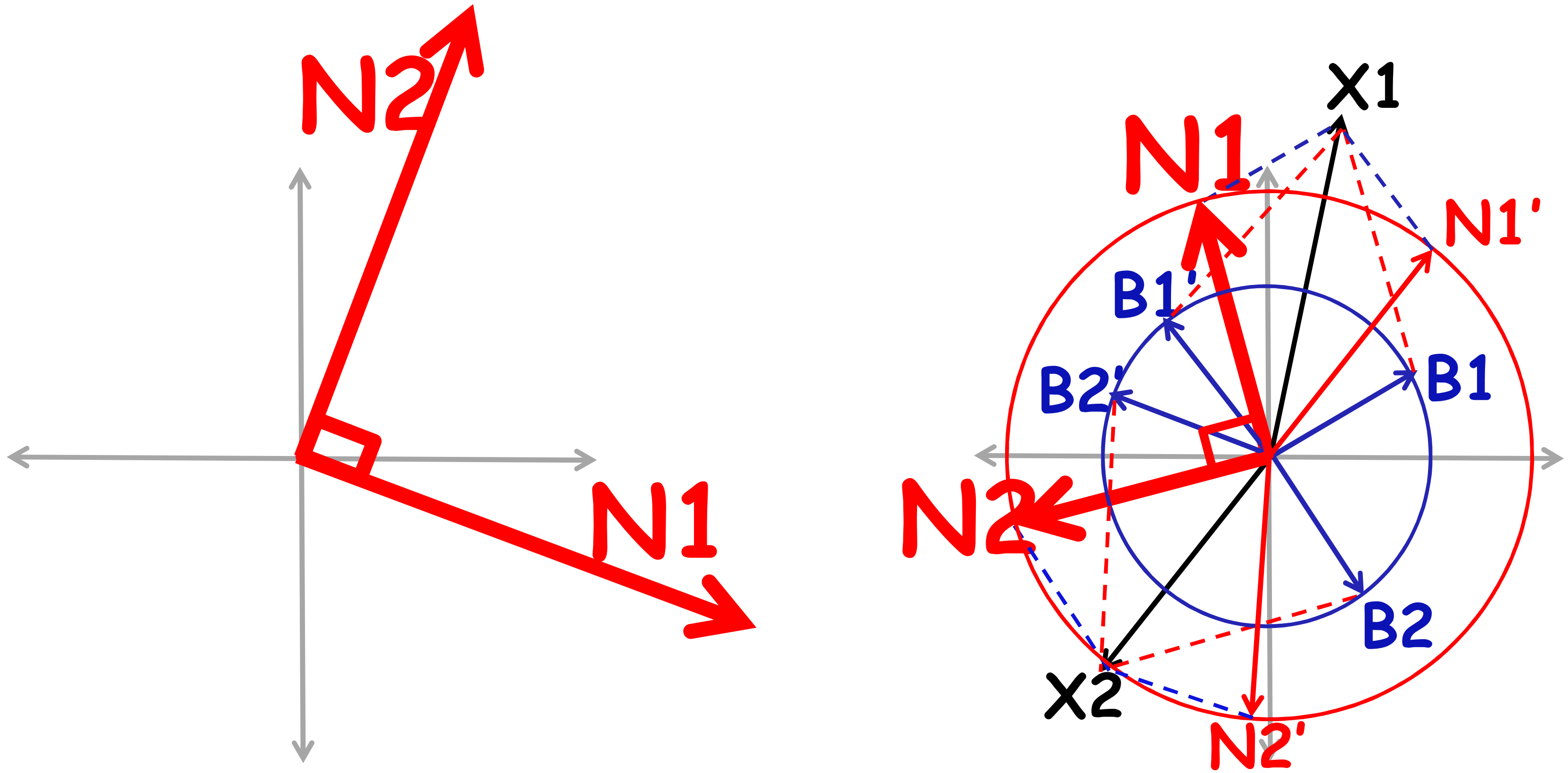
- What does Alice know?





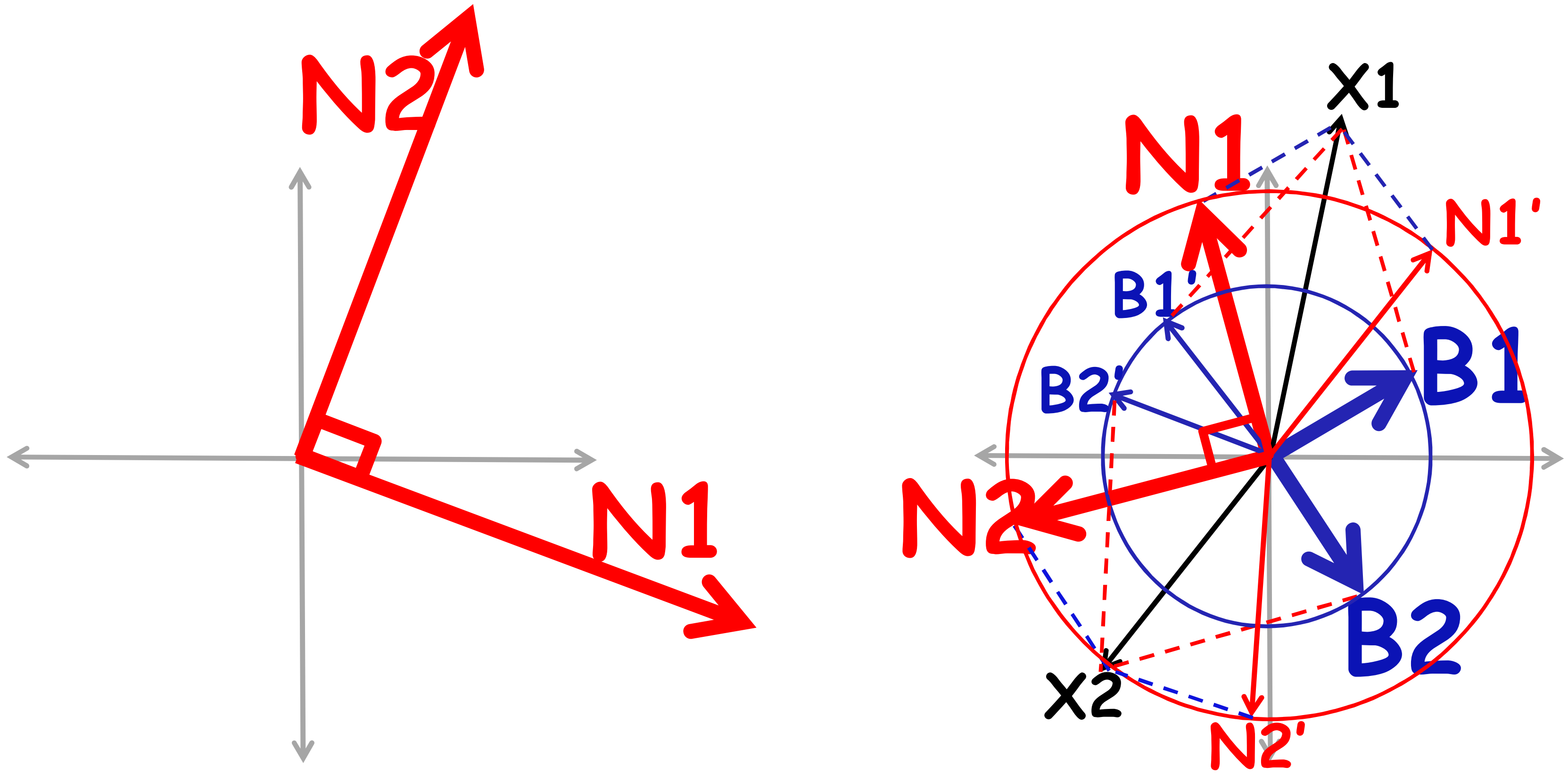
Pick the correct angle → +90 degrees

- What does Alice know?



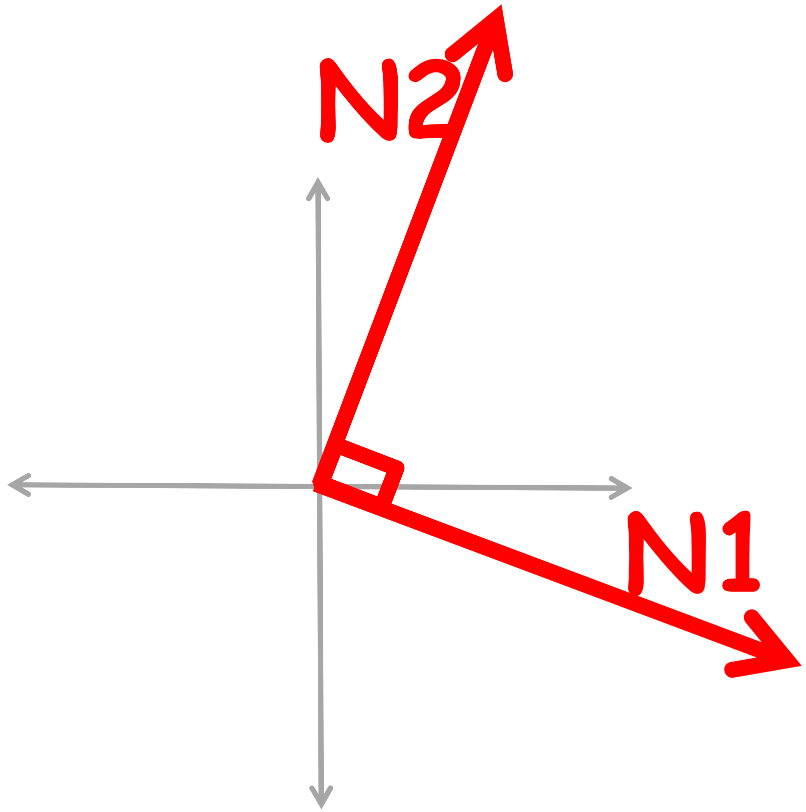
Pick the correct angle → +90 degrees

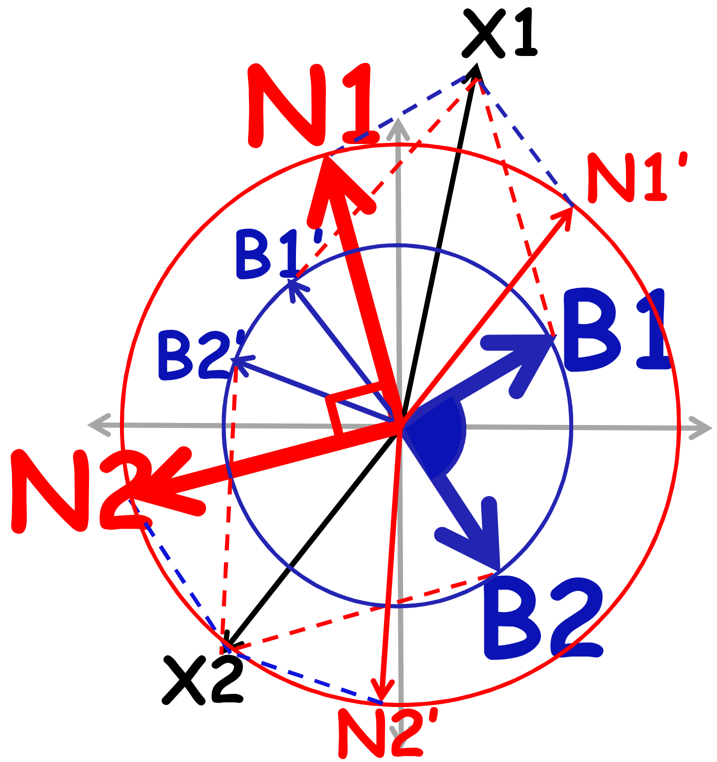
- What does Alice know?



Dictates solution for Bob's complex samples!

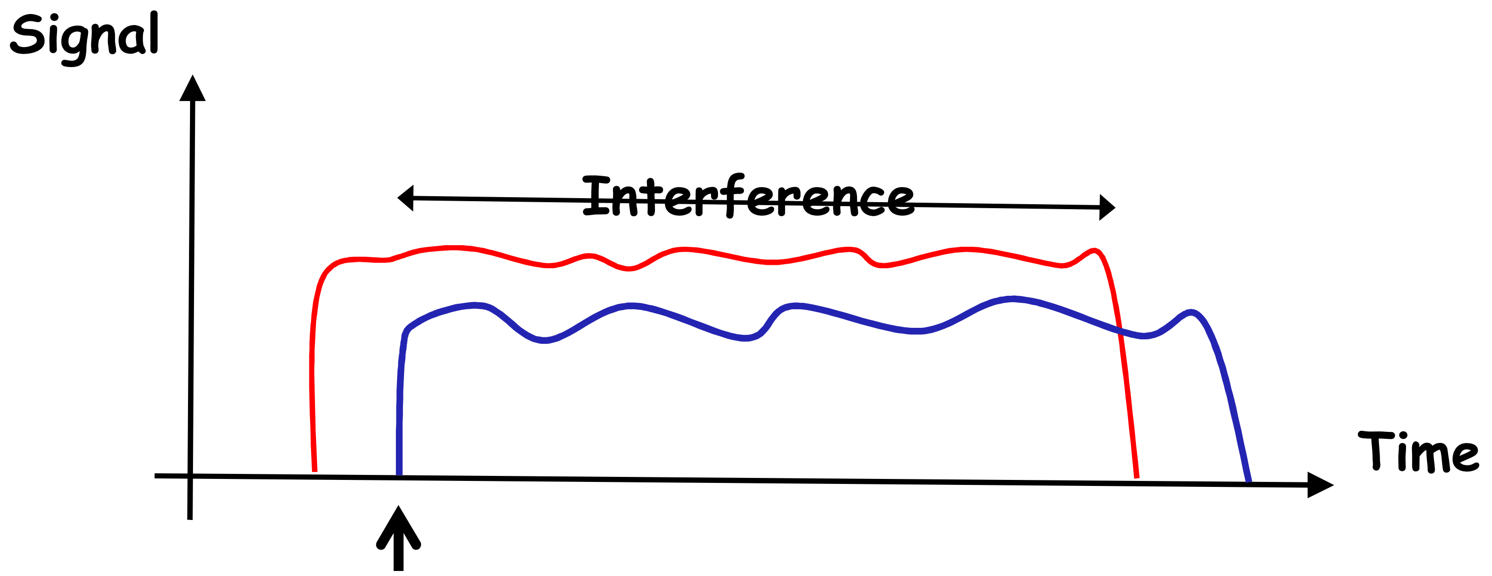
- What does Alice know?





- Alice finds angle between **B1** and **B2** and decodes

Decoding Algorithm – Decoding interference

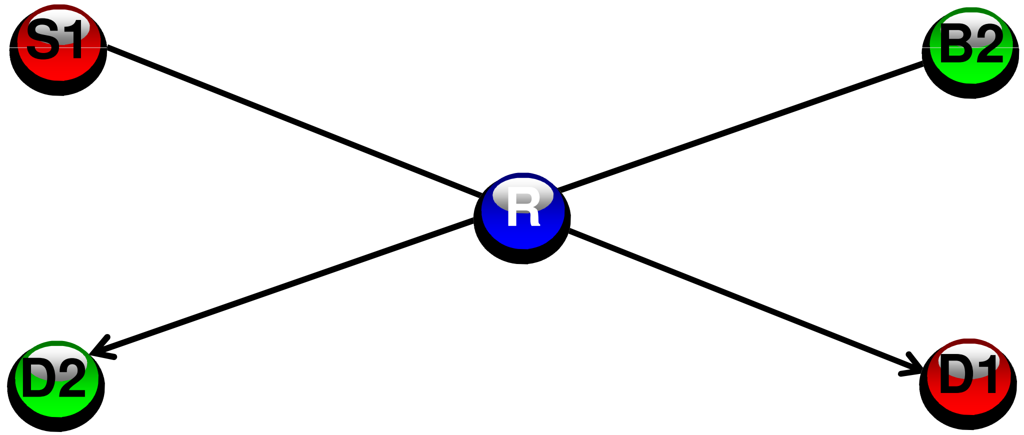


- Decode rest of the interfered part using this algorithm
- Decode final uninterfered part from Bob via standard GMSK demodulation
- Bob runs the same algorithm backwards

Generalizes to other topologies

- Chain (even with single flow)

- Cross



- Other COPE topologies

Summary

- Network coding allows routers to mix packet content before forwarding
- Inter-flow network coding mixes packets across flows
  - Exploits broadcast
  - Provides in-network compression
  - Exploits strategic collisions
- Prototypes that yield large throughput increases